Calibrating a reference scenario for a multi-region, recursive dynamic world CGE model

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May 2015

partnership for economic policy pep

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ABSTRACT

We describe how we developed a reference scenario for the PEP-w-t standard model (http://www.pep-net.org/pep-standard-cge-models), based on projections by Fouré, Bénassy-Quéré and Fontagné (2012, hereafter FBQF). FBQF propose a long-run growth scenario for 147 countries relying on the model MaGE, based on a three-factor production function of labour, capital and energy, plus two forms of technological progress. Given the great differences between FBQF's aggregate production function and the structure of PEP-w-t, we cannot incorporate all components of their projections into our model; we retain their projections of population, active population, real GDP and saving rates. Our reference scenario is calibrated by running a modified version of PEP-w-t which is constrained to follow FBQF real GDP projections, with total factor productivity (TFP) endogenous. To assess the acceptability of such an artifact, we examine the trajectories of variables that are common to PEP-w-t and FBQF projections, particularly the stock of capital and TFP. Despite radical differences between FBQF's aggregated model and PEP-w-t's CGE approach (where capital accumulation is endogenous, not projected), we have successfully aligned our reference scenario with FBQF's. Our comparison shows that the differences are sufficiently moderate to make our reference scenario acceptable and usable as a basis for policy and other simulations. Finally, we apply the growth accounting methodology to our reference scenario to confirm its consistency and to display its salient features.

INTRODUCTION

This document describes the method that was applied to develop a reference scenario for the PEP-w-t standard model (Robichaud *et al.*, 2013)[1]. Version 4.1 of PEP-w-t has been developed and tested using two aggregation schemes of the GTAP 8.1 database: a 14-region, 4-commodity aggregation, and a 25-region, 18-commodity one[2]. Appendix A contains a map and a list of the 25 regions, and Appendix B presents a list of the 18 commodities.

We base our reference scenario on the 1980-2050 historical data and projections developed by Fouré, Bénassy-Quéré and Fontagné (2012, 2013; hereafter FBQF) using the MaGE model[3]. Quoting from the non technical summary of their 2012 paper:

> "Based on a three-factor production function of labour, capital and energy, plus two forms of technological progress, we propose a long-run growth scenario for 147 countries and a time horizon of 2050 relying on the model MaGE (Macroeconometrics of the Global Economy). Our model is fitted with United Nations and International Labour Office labour projections, and econometric estimations of (i) capital accumulation, (ii) savings rate, (iii) relationship between savings and investment rate, (iv) education, (v) female participation, and (vi) technological progress (which includes energy and total factor productivity). Our study provides five novelties. First, we account for energy constraints by including its consumption in the production function and by taking account of rents accruing to oil exporting countries. Second, we estimate a non-unitary relationship between savings and investment, departing from assumptions of either a closed economy or full capital mobility. Third, we model female participation rates consistently with education catch-up. Fourth, we account for the 2008-09 global crisis by initialising our projection model in 2013 while relying on IMF short-term forecasts between 2010 and 2012. Finally, we disentangle real gross domestic product (GDP) growth rates from relative price effects through a consistent Balassa-Samuelson effect."

Given the great differences between the specification of the aggregate production function used by FBQF and the structure of production in PEP-w-t, we cannot incorporate all the components of their projections into our model. In addition, capital accumulation in PEP-w-t is endogenous, not projected. Specifically, we retain from FBQF their projections of population, labor force (active population), real GDP and saving rates.

Our reference scenario is calibrated by running a modified version of PEP-w-t which is constrained to follow FBQF real GDP projections, and where total factor productivity (TFP) is endogenous. In what follows we call this modified version of PEP-w-t the "baseline calibrating version". In the baseline calibrating version, labor supply and aggregate domestic saving rates are also set according to FBQF

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[1] http://www.pep-net.org/pep-standard-cge-models

[2] The first was already distributed with version 4.0 of PEP-w-t, with kind permission from GTAP. The 25-18 data file is not publicly distributed, in compliance with the GTAP licence under which it was constructed. Users who have a GTAP licence can easily reproduce it by following the instructions laid out in Robichaud (2014). And of course, users can build a data file with their own aggregation scheme.

[3] http://www.cepii.fr/CEPII/fr/bdd_modele/presentation.asp?id=13

projections. The paths of other exogenous model variables are either fixed on the basis of FBQF projections, or determined according to proportionality rules relative to other model variables. Capital accumulation, however, remains endogenous and does not follow FBQF. The baseline calibrating solution value of total factor productivity and other exogenous variables (including savings rates) constitute our reference scenario. Once calibrated, these variables are exogenously fixed at their calibrated values in the model.

The rest of this paper is organized as follows. Section 1 presents the calibration procedure in more detail. Section 2 compares the resulting reference scenario with FBQF projections. Section 3 applies growth decomposition to the reference scenario. A brief conclusion rounds off the paper.

1. CALIBRATION PROCEDURE

1.1 Total factor productivity

The first step in the specification of the baseline calibrating version of PEP-w-t consists in making productivity endogenous, while constraining real GDP growth rates exogenously to reproduce FBQF projections. This is a common approach in CGE models.

In PEP-w-t, each industry's value added consists of composite labor and composite capital, following a constant elasticity of substitution (CES) specification, to which we add a multifactor productivity multiplier:

$$VA_{j,z,t} = A_{z,t}^{VA} \, B_{j,z}^{VA} \left[\beta_{j,z}^{VA} LDC_{j,z,t}^{-\rho_{j,z}^{VA}} + \left(1 - \beta_{j,z}^{VA}\right) KDC_{j,z,t}^{-\rho_{j,z}^{VA}} \right]^{-\frac{1}{\rho_{j,z}^{VA}}} \qquad [001]$$

where

$VA_{j,z,t}$: Value added of industry *j* in region *z* in period *t*

$KDC_{j,z,t}$: Demand for composite capital by industry *j* in region *z* in period *t*

$LDC_{j,z,t}$: Demand for composite labor by industry *j* in region *z* in period *t*

$A_{z,t}^{VA}$: Multifactor productivity in region *z* in period *t*

$B_{j,z}^{VA}$: Scale parameter (CES – value added)

$\beta_{j,z}^{VA}$: Share parameter (CES – value added)

$\rho_{j,z}^{VA}$: Elasticity parameter (CES – value added); $-1 < \rho_{j,z}^{VA} < \infty$

$A_{z,t}^{VA}$ is endogenous in the baseline calibrating version of the model, determined by the constraint on real GDP. This forces the model to track the evolution of GDP according to FBQF's projections. Once calibrated, successive values of $A_{z,t}^{VA}$ are fixed in regular simulation runs of the model, and GDP is endogenous.

1.2 Savings rate

In the course of developing PEP-w-t and its reference scenario, it was observed that capital accumulation in PEP-w-t led to faster growth than projected by FBQF. Since our model is savings-driven, we found that this was largely due to high savings rates, which translate into high investment rates and rapid capital accumulation. With a neo-classical full-employment closure, rapid capital accumulation generates rapid GDP growth. To bring our reference scenario closer to FBQF projections, and to reduce the adjustment burden imposed on total factor productivity, we constrain savings rates in that scenario to evolve through time according to savings rate variations projected by FBQF (initial savings rates, of course, are bound by the calibration procedure to the GTAP database values.

Savings rates projections by FBQF rely on the Masson, Bayoumi and Samiei (1998) life-cycle approach, already employed in Poncet (2006). “In this approach, the gross savings rate depends on the age-structure of the population and the GDP-per-capita gap with the leading [US] economy.” (FBQF, 2012, p. 22). FBQF savings rates projections are ratios of aggregate domestic savings over GDP. To make savings rates in our reference scenario follow the evolution of FBQF projections, we rearranged the way savings are determined in the baseline calibrating version of PEP-w-t.

In versions of PEP-w-t older than 4.0, there were three components of domestic savings: depreciation, government savings and household savings. The determination of depreciation is unchanged relative to versions of PEP-w-t before 4.0: it is the product of the price of capital, the rate of depreciation and the stock of capital. But in the current version, instead of being directly transferred to savings, depreciation is paid to households as part of their (gross) income from capital (households are the sole recipients of capital income).

Household savings in PEP-w-t are a linear function of disposable income, which allows for the marginal propensity to save to be different from the average propensity.

$$SH_{z,t} = PIXCON_{z,t}^{\eta} sh0_{z,t} + sh1_{z,t} YDH_{z,t} \qquad [002]$$

where

$PIXCON_{z,t}$: Consumer price index in region z in period t

$SH_{z,t}$: Household savings in region z in period t

$YDH_{z,t}$: Household disposable income in region z in period t

η : Price-elasticity of the savings function intercept

$sh0_{z,t}$: Intercept in period t (household savings)

$sh1_{z,t}$: Slope in period t (household savings)

This choice was originally made in the specification of the single-country PEP-1-1 model, motivated by the fact that it is common for certain household categories to have negative savings; in that case, the traditional formulation, where savings are a fixed proportion of income, has the undesirable propertiy that *dis*saving increases as income grows.

Regarding government savings, it must be remembered that the GTAP database is structured according to the single regional agent paradigm. In designing and calibrating PEP-w-t, we have chosen to extract tax receipts and public expenditures from the regional agent's income and expenditures, and so to distinguish two regional agents, government and households[4]. We shall see that the way savings are determined in the baseline calibrating version of PEP-w-t is closer to the single-agent paradigm.

We now detail the determination of savings in the baseline calibrating version of PEP-w-t. First of all, the marginal savings rate is initially set equal to the ratio of domestic savings over GDP:

$$sh1_z^O = \frac{SH_z^O + SG_z^O}{GDP_z^{IB.O}} \qquad [003]$$

where superscript *O* designates benchmark values derived from GTAP data, and

$GDP_{z,t}^{IB}$: GDP at market prices (income-based) in region z in period t

$SG_{z,t}$: Government savings in region z in period t

The numerator of [003] reflects the fact that depreciation is paid to households as part of their (gross) income from capital. The choice of GDP_z^{IB-O} in the denominator of 003 is motivated by the fact that

$$GDP_{z,t}^{IB} = YDH_{z,t} + YG_{z,t} \qquad [004]$$

where $YG_{z,t}$ is total government income in region z in period t.

So $sh1_z^O$ is a weighted average of household and government savings rates, which is consistent with the single-agent paradigm.

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[4] Note, however, that government transfers to households are totally absent from both the GTAP database and PEP-w-t. Consequently, government surpluses or deficits in the PEP-w-t benchmark data are narrowly defined as tax receipts minus current public expenditures.

Next, starting with $sh1_z^O$, exogenous marginal savings rates are determined from FBQF projections using the recursive formula

$$sh1_{z,t+1} = \left(1 + g_{z,t}^{sh1}\right) sh1_{z,t} \qquad [005]$$

where $g_{z,t}^{sh1}$ is the projected rate of change of the domestic savings ratio from t to t+1 according to FBQF.

The following step is to determine the intercept of the household savings function using the baseline calibrating version of PEP-w-t, in which we introduced an extra equation which constrains domestic savings to evolve relative to GDP following equation [005]:

$$SH_{z,t} + SG_{z,t} = sh1_{z,t} GDP_{z,t}^{IB} \qquad [006]$$

Government savings are determined endogenously elsewhere in the model: $SG_{z,t}$ is the difference between endogenous tax receipts and the exogenously determined amount of public expenditures. So household savings $SH_{z,t}$ are determined by equation [006]. With the addition of an extra equation, intercepts $sh0_{z,t}$ are left endogenous; they are determined by equation [007].

$$SH_{z,t} = PIXCON_{z,t}^{\eta} sh0_{z,t} + sh1_{z,t} YDH_{z,t} \qquad [007]$$

which, combined with [006], amounts to

$$sh0_{z,t} = \frac{sh1_{z,t}\left(GDP_{z,t}^{IB} - YDH_{z,t}\right) - SG_{z,t}}{PIXCON_{z,t}^{\eta}} \qquad [008]$$

Finally, the solution values of $sh0_{z,t}$ for the baseline calibrating version of PEP-w-t are used as the exogenous household savings function intercepts in the regular version of the model.

It may be interesting to examine the calibration procedure just described from a slightly different perspective. Equation [006] is equivalent to

$$SH_{z,t} = sh1_{z,t} GDP_{z,t}^{IB} - SG_{z,t} \qquad [009]$$

$$SH_{z,t} = sh1_{z,t}\left(YDH_{z,t} + YG_{z,t}\right) - SG_{z,t} \qquad [010]$$

$$SH_{z,t} = sh1_{z,t} YDH_{z,t} + \left(sh1_{z,t} YG_{z,t} - SG_{z,t}\right) \qquad [011]$$

Equation [011] says that household save a fraction $sh1_{z,t}$ of their own disposable income, plus any shortcoming of government savings relative to the same fraction of government income. One interpretation of this is that there is no fiscal illusion in the way households save[5]. It is also remindful of the so-called Kaldor closure in CGE models, where savings are forced to adjust endogenously to finance exogenously determined investments; indeed, although investments remain endogenous, they are determined by domestic savings, an exogenous fraction of the endogenous GDP, and foreign savings, equal to minus the current account balance, which also follows an exogenous path. If such a pseudo-Kaldor closure were maintained in the standard version of the model, changes in the government fiscal position (surplus or deficit) would be automatically neutralized by an adjustment of opposite sign in household savings.

1.3 Exogenous growth factors and exogenously growing variables

Population and LES minimum consumption

Population is not explicitly present in PEP-w-t. But population growth projected by FBQF drives the evolution of minimum consumption $C_{i,z,t}^{MIN}$ in the linear expenditure system (LES) of household consumption demand.

Labor supply

Quite sensibly, labor supply evolves according to economically active population growth rates projected by FBQF.

Exogenous growth factor exogro *and the intercept of the income tax function*

We compute an exogenous growth factor – called *exogro* – which combines projected labor force evolution and a moving average of the projected growth rate of output per worker. We use *exogro* for two purposes. First, we use it to update the value of the intercept of the household income tax function, specified as

$$TDH_{z,t} = PIXCON_{z,t}^{\eta} ttdh0_{z,t} + ttdh1_{z,t} YH_{z,t} \quad [012]$$

where

$TDH_{z,t}$: Household income taxes in region *z* in period *t*

$ttdh0_{z,t}$: Intercept (household income taxes) in region *z* for period *t*

5 Empirical research, however, indicates that households do not offset changes in government savings completely. Masson *et al*. (1998) conclude that "there seems to be a substantial offset of changes in the government fiscal position from private saving, averaging 75 percent, depending on whether those changes are due to changes in government spending or in taxes." (p. 497).

$ttdh1_{z,t}$: Marginal household income tax rate in region *z* for period *t*

$YH_{z,t}$: Total household income in region *z* in period *t*

Intercept $ttdh0_{z,t}$ is a pseudo-volume variable, in that it is partially or fully indexed, according to whether price elasticity η is less than or equal to 1.

Secondly, *exogro* is used to compute the current account balance index $cabix_{z,t}$ (see below). Details of the computation of *exogro* are found in Appendix C.

1.4 Current account balance

In the PEP-w-t default model closure, real current account balances in terms of the international currency are fixed exogenously[6]. They are defined as

$$CABX_{z,t} = \frac{CAB_{z,t}}{e_{z,t} PIXGDP_t^W} \quad [013]$$

where

$CABX_{z,t}$: Real period *t* current account balance of region *z* in terms of the international currency

$CAB_{z,t}$: Current account balance of region *z* in period *t*, expressed in region *z*'s currency

$e_{z,t}$: Exchange rate; price of international currency in terms of region *z*'s local currency in period *t*

$PIXGDP_t^W$: World GDP deflator in period *t*

In view of the equilibrium condition that the worldwide sum of current account balances expressed in the common international currency must be zero,

$$\sum_z CABX_{z,t} = 0 \quad [014]$$

fixing all $CABX_{z,t}$ but one implicitly fixes the remaining one (a form of Walras' Law)[7].

However, it would be incorrect to fix all $CABX_{z,t}$ but one by making them evolve following each region's exogenous growth factor $exogro_{z,t}$. For if every region's $CABX_{z,t}$ grows exogenously according to

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6 The reasons for fixing *CABX* rather than *CAB* are discussed in depth in Lemelin (2015).

7 This remains true even though the equilibrium condition is redundant, given PEP-w-t equations 33, 34 and 81-83 (see Robichaud *et al*. 2013, p. 43, and Lemelin, 2015).

$exogro_{z,t}$, then the worldwide sum of $CABX_{z,t}$ over z will not remain at zero – an equilibrium condition –, unless $exogro_{z,t}$ is equal across regions. And since, in fact, one region's $CABX_{z,t}$ is left implicit due Walras's Law, then that region's implicit $CABX_{z,t}$ will not in general grow according to its $exogro_{z,t}$. Consequently, if all $CABX_{z,t}$ but one grow following $exogro_{z,t}$, then the model solution depends on the choice of which region's $CABX_{z,t}$ remains implicit, a violation of model homogeneity.

To make the model solution independent of the choice of which region's $CABX_{z,t}$ remains implicit, the $CABX_{z,t}$ follow a path defined by a specific index, $cabix_{z,t}$ (CAB IndeX), which is formed from $exogro_{z,t}$ by inflating and deflating proportionately the updating factors of positive and negative current account balances in such a way that their sum remains zero. Exogenous real current account balances in terms of the international currency are then determined from

$$CABX_{z,t} = cabix_{z,t} CABX_{z}^{O} \qquad [015]$$

The detailed procedure to compute $cabix_{z,t}$ is presented in Appendix D.

1.5 Government spending and public investment

In the baseline calibrating version of PEP-w-t, we make nominal government expenditures a constant share of the reference scenario nominal GDP. In a previous version, real government current expenditures were fixed in proportion to real GDP. The latter specification, however, proved incompatible with FBQF projections: the model crashed due to the combination, in Russia, of a projected fall in the labor force, and a projected quintupling of real GDP[8].

Compared to maintaining a constant ratio of real government spending to real GDP, fixing nominal government expenditures as a constant share of the reference scenario nominal GDP is analogous to switching from a Leontief allocation of GDP to a Cobb-Douglas one: as the price index of the labor-intensive government services industry rises or falls relative to the general GDP deflator, the ratio of real government expenditure to real GDP falls or rises in the same proportion. Once the evolution of nominal current government expenditures and of the corresponding price index have been determined in the baseline calibrating version of the model, the corresponding path of real government expenditures is included in the reference scenario.

The reference scenario calibration of government expenditures is achieved by the addition of an extra equation to the model:

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8 A detailed analysis of the crash scenario is given in Appendix D of Lemelin and Robichaud (2014).

$$G_{z,t} = G_z^O \frac{GDP_z^{BP}}{GDP_z^{BP.O}} \qquad [016]$$

where

$G_{z,t}$: Current government expenditures on goods and services in region z in period t

$GDP_{z,t}^{BP}$: GDP at basic prices in region z in period t

and where superscript O designates benchmark values derived from GTAP data.

The reference scenario values of $G_{z,t}$ are converted to real government expenditures using

$$G_{z,t}^{REAL} = \frac{G_{z,t}}{PIXGVT_{z,t}} \qquad [017]$$

where

$G_{z,t}^{REAL}$: Real government expenditures in region z in period t

$PIXGVT_{z,t}$: Public expenditure price index in region z in period t

The reference scenario solution values of $G_{z,t}^{REAL}$ then enter the regular model as exogenous.

Of course, this is a real modification relative to the PEP-w-t version 4.0 reference scenario described in Appendix D of Robichaud *et al*. (2013), where real government expenditures were assumed to grow exogenously at a rate reflecting the expected growth of population and GDP per capita (exogenous growth factor *exogro* described above). In the current reference scenario, real government expenditures evolve independently of population. In the special case where the price indexes of GDP and of government expenditures remain equal, *real* government expenditures grow at the same rate as real GDP, regardless of how GDP per capita evolves. Moreover, in the general case where the price indexes do *not* remain equal, real government expenditures may grow faster, or slower than real GDP. Initially, that change was introduced as an *ad hoc* mechanism so that the model could be solved. But in hindsight, we feel that the new specification can be justified on the grounds that in reality, public finances are indeed under constraint, and the financial constraints are largely defined in nominal terms.

Public investment expenditures are treated in the same way as current government spending. Nominal public investment expenditures are determined in the baseline calibrating version of PEP-w-t as a constant proportion of nominal GDP, using the following calibrating equation:

$$PK_{z,t} IND_{k,pub,z,t} = PK_z^O IND_{k,pub,z}^O \frac{GDP_z^{BP}}{GDP_z^{BP.O}} \quad [018]$$

where

$IND_{k,j,z,t}$: Volume of new type *k* capital investment to industry *j* in region *z* in period *t*

$PK_{z,t}$: Price of new private capital in region *z* in period *t*

The reference scenario values of $IND_{k,pub,z,t}$ then enter the regular model as exogenous.

1.6 Baseline Calibrating summary

To summarize, the baseline calibrating version of PEP-w-t has exogenous domestic savings rates, which implicitly determine household savings and their intercept, and it is constrained to follow FBQF's GDP growth paths, with endogenous total factor productivity factors $A_{z,t}^{VA}$. The baseline calibrating version is run to the 2050 horizon, and the solution values of $A_{z,t}^{VA}$ and $sh0_{z,t}$ are retrieved to be applied respectively as exogenous total factor productivity indexes and exogenous household savings function intercepts in the standard version of the model. With these devices, absent any exogenous shock, PEP-w-t in its standard version tracks FBQF's GDP growth projections.

1.7 What to do about the 2008-2009 crisis?

FBQF write :

> « Using MaGE allows us to make long-run economic projections for 147 countries. It relies on the assumption that in the long run, only supply-side factors (labour, capital, TFP, energy productivity) matter for economic growth. Thus, the starting point for the projections should be a year when GDP was at its potential level in most countries. However, we are left with the problem of the 2008-09 global crisis. We could take as our starting point a pre-crisis year – say 2007, but the crisis involved falls in production and income and also a collapse in investments, which is likely to have a long-lasting impact on potential output. Therefore, we (i) rely on IMF GDP forecasts (Autumn 2011) to 2012, (ii) assume that the output gap has been closed at that date, (iii) adjust TFP levels accordingly, based on factor projections during the crisis, and (iv) perform GDP projections for 2013 to 2050. This methodology may overstate the drop in TFP during the crisis since we are unable to account for the temporary fall in investment rates and the rise in unemployment, whose effects could extend beyond 2012. However, this feature is benign since our interest is in GDP, not employment or TFP. » (p. 41)

Of course, since our reference scenario follows FBQF, the 2008-2009 crisis is implicitly handled in the same way. However, even though PEP-w-t is calibrated on GTAP 8.1 data for 2007, that particular reference year does not seem to be characterized by the collapse in investments mentioned by FBQF.

Indeed, as we shall see, the TFP improvement implicit in the evolution of $A_{z,t}^{VA}$ to track FBQF's GDP growth projections are less than FBQF TFP projections imply.

2. PEP-w-t REFERENCE SCENARIO AND FBQF PROJECTIONS

The calibration of $A_{z,t}^{VA}$ to track FBQF's GDP growth projections is clearly an artifact that lays open to criticism. In particular, TFP growth just "happens" in the model, as if it were heaven-sent. In the real world, as we all know, productivity improvements are only achieved from costly research and development (R&D), investment in human capital (education), and in physical capital that embodies more advanced technology. There is no relationship in the model between the cost of improving productivity and TFP growth. Also, the way $A_{z,t}^{VA}$ appears in the value added production function implies that productivity growth is Hicks-neutral. This ignores the possibility that higher educational attainment, and more generally the accumulation of human capital, increase labor effectiveness, and generate labor-augmenting (Harrod-neutral) productivity gains[9]. So the labor-augmenting effects of human capital accumulation are subsumed in TFP, and this is not without consequence for the validity of the growth decomposition exercise performed below.

Because we are aware of these and other weaknesses, we examine the trajectories of variables that are common to the PEP-w-t reference scenario and the FBQF projections: GDP, labor, capital and TFP. Since the first two follow FBQF by design, our attention focuses on capital and TFP. Given the great differences between the specification of the aggregate production function used by FBQF and the structure of production in PEP-w-t, we expect to find substantial differences in the trajectories followed by the two variables of interest. In our opinion, our comparison shows that the differences, while substantial, are sufficiently moderate to make our reference scenario acceptable and usable as a basis for policy and other simulations.

Table 1 reports the calibrated values of $A_{z,t}^{VA}$ for year 2050, together with FBQF's projected TFP index (2007 = 1).

[9] FBQF project educational attainment, but their model does not include a labor-augmenting productivity factor. Rather, educational attainment is used to project female labor force participation, and tertiary education is used in the econometric estimation of TFP evolution.

Table 1 – Calibrated $A_{z,t}^{VA}$ and TFP according to FBQF, year 2050

	TFP	$A_{z,t}^{VA}$	$A_{z,t}^{VA}$ **/ TFP**
01-Oceania	1.77	1.34	0.76
02-ChinaHK	4.08	3.48	0.85
03-Japan	1.74	1.35	0.78
04-RoAsIC	2.15	1.80	0.84
05-AsiaDev	2.23	1.61	0.72
06-India	3.84	2.26	0.59
07-CSAsDev	4.02	2.15	0.53
08-USA	1.40	1.20	0.86
09-Mexico	1.95	1.56	0.80
10-RoNA	1.70	1.34	0.79
11-CAmCar	2.58	1.73	0.67
12-Brazil	2.11	2.14	1.02
13-RoLAm	2.03	1.82	0.90
14-France	1.49	1.22	0.82
15-Germany	1.59	1.50	0.94
16-Italy	1.43	1.08	0.76
17-UK	1.48	1.22	0.83
18-REuroZ	1.64	1.31	0.80
19-Russia	4.06	1.53	0.38
20-RoEEurp	3.12	2.25	0.72
21-RoEurope	1.43	1.30	0.91
22-Turkey	2.51	1.61	0.64
23-MENA	1.51	1.44	0.96
24-SAfrica	1.92	1.72	0.90
25-RoAfrica	2.49	3.62	1.46

First of all, we note that $A_{z,t}^{VA}$ is greater than 1 for all regions. This means that, if PEP-w-t were run with all $A_{z,t}^{VA}$ fixed at 1 rather than at their calibrated values, the GDP solution values would not overshoot the FBQF projections. Put otherwise, this implies that if we interpret the calibrated $A_{z,t}^{VA}$ as TFP, tracking FBQF's GDP growth projections does not require a decline in productivity, which is reassuring. Next, for 19 regions out of 25, the value of $A_{z,t}^{VA}$ is between 1 and 2, and only in the cases of China and the Rest of Africa does it exceed 2.26. Finally, except for the Rest of Africa and, marginally, Brazil, all $A_{z,t}^{VA}$ are less than FBQF's projected TFP index. This means that if $A_{z,t}^{VA}$ were made to evolve as FBQF's TFP, the reference scenario generated by PEP-w-t would involve stronger growth than FBQF's projections, except for the Rest of Africa. This of course calls for prudence if $A_{z,t}^{VA}$ is interpreted as a TFP index, since it is in disagreement with FBQF projections.

Table 2 – Index of capital stock in PEP-w-t and FBQF projections, year 2050

	FBQF	PEP-w-t	Ratio PEP/FBQF
01-Oceania	2.68	3.88	1.45
02-ChinaHK	11.90	10.18	0.86
03-Japan	1.49	2.08	1.40
04-RoAsIC	4.52	4.20	0.93
05-AsiaDev	5.07	6.05	1.19
06-India	9.27	13.07	1.41
07-CSAsDev	7.24	8.14	1.12
08-USA	1.70	2.78	1.64
09-Mexico	3.22	2.84	0.88
10-RoNA	2.27	3.41	1.51
11-CAmCar	5.57	5.82	1.04
12-Brazil	3.42	2.40	0.70
13-RoLAm	3.79	3.07	0.81
14-France	1.55	2.18	1.41
15-Germany	1.20	1.20	1.00
16-Italy	0.96	1.46	1.53
17-UK	1.86	2.74	1.48
18-REuroZ	1.75	2.48	1.42
19-Russia	3.85	10.47	2.72
20-RoEEurp	3.25	4.23	1.30
21-RoEurope	1.86	2.17	1.17
22-Turkey	4.39	5.41	1.23
23-MENA	4.51	3.80	0.84
24-SAfrica	3.64	2.98	0.82
25-RoAfrica	8.55	3.28	0.38

Now consider capital accumulation. Table 2 displays a Fisher quantity index of the stock of capital in 2050 according to PEP-w-t's reference scenario, compared to an index of capital stock according to FBQF projections. Clearly, in spite of making savings rates evolve in line with the FBQF scenario, capital accumulation generated by PEP-w-t is at variance with FBQF projections. Moreover, it appears that deviations of the calibrated $A_{z,t}^{VA}$ from TFP tend to compensate for discrepancies in capital accumulation. This is illustrated in Figure 1 below.

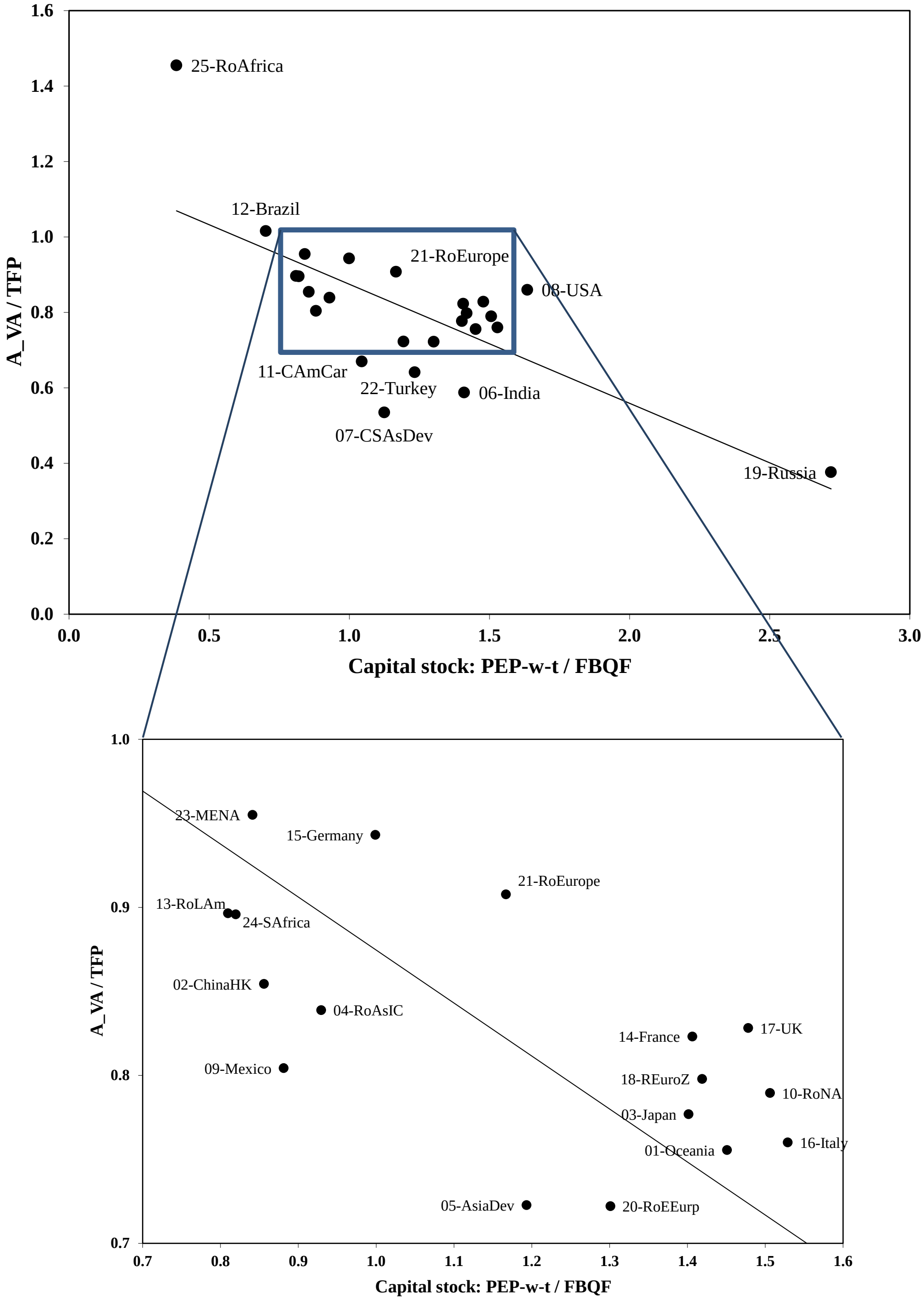
There is a –0.719 correlation between: (1) the ratio of $A_{z,t}^{VA}$ over TFP, and (2) the ratio of capital stock in the PEP-w-t baseline simulation and in FBQF projections. When the two outliers are excluded (Rest of Africa at the top, and Russia to the right), the correlation is still –0.630. The general conclusion that can be drawn from Figure 1 is that, in the PEP-w-t reference scenario, endogenous capital accumulation tends to be faster than projected by FBQF, which is offset by values of $A_{z,t}^{VA}$ that are less than would be expected from FBQF's projection of TFP.

But short of tinkering arbitrarily with model parameters (such as A_z^K, the scale parameter of the new capital implicit production function), it is difficult to see how PEP-w-t could be brought closer to the FBQF projections.

Some aspects of the comparison between our reference scenario and FBQF's projections remain intriguing, however. In particular, consider the case of regions such as Mexico, China or South Africa in Figure 1. They show an apparent contradiction: while (1) $A_{z,t}^{VA}$ is less than FBQF's projected TFP, and (2) the capital stock index is less according to the PEP-w-t reference scenario than in FBQF projections, these regions nevertheless track FBQF's GDP projections. This is out of line with the productivity-capital accumulation trade-off mentioned above. The source of this paradox cannot be labor supply, as our reference scenario follows FBQF in that respect. A possible explanation might be that FBQF take into account energy use and efficiency, while PEP-w-t does not. Or perhaps the explanation lies in the way the evolution of current account balances is modelled. But that puzzle will remain unsolved for the time being.[10]

[10] In private communication, in December 2014, Jean Fouré wrote: "I ran a quick check, and it seems to me that the paradox you mention between capital accumulation and TFP cannot come mainly from our taking into account energy in our production function: although it is true that South Africa and China are indeed not very efficient in their use of energy (taking into account energy has a strong impact on GDP – between 7 and 10 percent), such is not really the case with Mexico (2%). I would be inclined [to think that the source of the paradox is to be found] in the direction of current account balances, which evolve in a totally different manner in our respective models (re-equilibration [of the Chinese balance] is particularly quick in ours)." (authors' translation from French).

Figure 1 – TFP ratios and capital stock index ratios in PEP-w-t and FBQF projections, year 2050



3. GROWTH DECOMPOSITION APPLIED TO THE PEP-w-t REFERENCE SCENARIO

The growth decomposition methodology is described in Appendix E. To validate the consistency of our reference scenario, we computed the real GDP that would have been produced without productivity growth, given the evolution of the aggregate stock of capital and labor supply, using the following recursive relationship:

$$\frac{GDP_{z,t}^{NoTFP}}{GDP_{z,t-1}^{NoTFP}} = \exp\left[\bar{s}_{LDC,z,t}\Delta \ln QIXLDC_{z,t} + \bar{s}_{KDC,j,z,t}\Delta \ln QIXKDC_{z,t}\right] \quad [019]$$

where

$GDP_{z,t}^{NoTFP}$: real GDP that would have been produced without productivity growth;

$QIXLDC_{z,t}$: Fisher quantity index of labor services

$\Delta \ln QIXLDC_{j,z,t} = \ln\left(QIXLDC_{j,z,t} / QIXLDC_{j,z,t-1}\right)$

$QIXKDC_{z,t}$: Fisher quantity index of capital services

$\Delta \ln QIXKDC_{j,z,t} = \ln\left(QIXKDC_{j,z,t} / QIXKDC_{j,z,t-1}\right)$

$s_{LDC,z,t}$: share of labor in nominal $GDP_{z,t}^{BP}$

$$\bar{s}_{LDC,z,t} = \frac{s_{LDC,z,t} + s_{LDC,z,t-1}}{2}$$

$s_{KDC,z,t}$: share of capital in nominal $GDP_{z,t}^{BP}$

$$\bar{s}_{KDC,z,t} = \frac{s_{KDC,z,t} + s_{KDC,z,t-1}}{2}$$

Finally, we compare $GDP_{z,t}^{BP-REAL} / GDP_{z,t}^{NoTFP}$ with the calibrated value of $A_{z,t}^{VA}$. If our growth decomposition were exact, we would expect

$$GDP_{z,t}^{BP-REAL} = A_{z,t}^{VA}\, GDP_{z,t}^{NoTFP} \quad [020]$$

And indeed, observed discrepancies for all periods and all regions vary between –0.749% and +0.405%. Given that the Fisher indexes of labor and capital are not “exact” indexes (in the sense of Diewert, 1976), this is a very close fit.

As demonstrated in Appendix E, combining equations [019] and [020] yields

$$\frac{GDP_{z,t}^{BP\_REAL}}{GDP_{z}^{BP\_REALO}} = \frac{A_{z,t}^{VA}}{A_{z}^{VAO}} \exp\left\{\sum_{\theta=t1+1}^{t}\left[\bar{s}_{LDC,z,\theta}\Delta \ln QIXLDC_{z,\theta}\right]\right\} \exp\left\{\sum_{\theta=t1+1}^{t}\left[\bar{s}_{KDC,z,\theta}\Delta \ln QIXKDC_{z,\theta}\right]\right\} \quad [021]$$

This decomposes the real GDP index into the product of three factors. The first is the calibrated change in $A_{z,t}^{VA}$, which may be interpreted as the change in TFP. The second and third factors are indexes of the contribution of labor and capital to growth; note that these factors depend on the evolution of both Fisher quantity indexes, and factor shares. Table 3 gives the value of the real GDP index, the labor factor, the capital factor, and the residual factor for 2050; it can be verified that the product of the latter three is very closely equal – virtually identical – to the real GDP index. The right-hand side of Table 3 presents the rank of each index among regions.

Table 3 shows that the labor factor is the one that contributes least to growth, in all cases. In 19 out of 25 regions, it is the capital factor which contributes most. And among the remaining 6 cases, there are only three for which the contribution of the residual (TFP) factor exceeds that of capital by more than 10% (they are: Brazil, Germany, and the Rest-of-Africa). Indeed, the rank-correlation of the capital factor to the GDP index is 0.915, while the rank-correlation of the TFP factor is 0.808; the rank-correlation of the labor factor, however, is only 0.564.[11] To summarize, capital and TFP account for GDP growth more than labor.

In reading Table 3, the reader must keep in mind the remarks made in Section 2. First, recall that the calibrated value of $A_{z,t}^{VA}$ is the TFP that we assign each region to make it follow FBQF's GDP projections. $A_{z,t}^{VA}$ differs from FBQF's TFP projections, and its contribution to growth is merely a reflection of the discrepancy between the trajectory generated by PEP-w-t with constant TFP and the trajectory projected by FBQF. Secondly, the labor factor takes no account whatsoever of changes in the quality of labor associated with education and the accumulation of human capital. In principle, labor quality improvements should enter the model as labor-augmenting (Harrod-neutral) productivity gains. But here, all productivity gains are assumed to be Hicks-neutral TFP gains that leave unchanged the relative marginal products of capital and labor. It follows that gains from education are subsumed in TFP, and the growth decomposition exercise presented here most certainly understates the contribution of *effective*

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[11] Correlations between factor values and the GDP index are: 0.568 for the labor factor, 0725 for the capital factor, and 0.852 for TFP.

labor (the quantity of labor corrected for quality changes).

To conclude, the growth decomposition presented in Table 3 is essentially a check of model consistency. It was performed in particular dissipate our own doubts regarding regions such as Mexico, China or South Africa, where (1) $A_{z,t}^{VA}$ is less than FBQF's projected TFP, and (2) the capital stock index is less according to the PEP-w-t reference scenario than in FBQF projections (Figure 1). The growth decomposition shows that indeed, the evolution of the stock of capital and of TFP in our reference scenario are consistent with that of GDP.

Table 3 – Contributions to GDP growth, year 2050

	GDP index and contributing factors				Ranks			
	GDP index	Labor factor	Capital factor	$A_{z,t}^{VA}$	GDP index	Labor factor	Capital factor	$A_{z,t}^{VA}$
01-Oceania	3.07	1.33	1.72	1.34	16	3	15	19
02-ChinaHK	10.11	0.90	3.20	3.48	3	20	3	2
03-Japan	1.65	0.87	1.39	1.35	23	23	21	17
04-RoAsIC	4.61	1.20	2.13	1.80	9	7	10	8
05-AsiaDev	5.29	1.15	2.86	1.61	6	12	5	12
06-India	10.95	1.28	3.79	2.26	2	5	1	3
07-CSAsDev	8.65	1.33	3.02	2.15	4	4	4	5
08-USA	1.92	1.17	1.36	1.20	19	9	23	24
09-Mexico	3.59	1.13	2.03	1.56	13	13	11	13
10-RoNA	2.45	1.11	1.64	1.34	17	15	16	18
11-CAmCar	6.24	1.38	2.61	1.73	5	2	7	9
12-Brazil	3.48	1.12	1.45	2.14	15	14	19	6
13-RoLAm	4.03	1.24	1.78	1.82	12	6	13	7
14-France	1.78	1.04	1.40	1.22	21	17	20	22
15-Germany	1.33	0.83	1.08	1.50	24	25	25	15
16-Italy	1.19	0.89	1.22	1.08	25	21	24	25
17-UK	1.97	1.09	1.47	1.22	18	16	18	23
18-REuroZ	1.91	0.95	1.54	1.31	20	19	17	20
19-Russia	5.07	0.89	3.76	1.53	7	22	2	14
20-RoEEurp	4.10	0.86	2.14	2.25	11	24	9	4
21-RoEurope	1.77	1.00	1.36	1.30	22	18	22	21
22-Turkey	4.97	1.15	2.68	1.61	8	11	6	11
23-MENA	4.18	1.19	2.43	1.44	10	8	8	16
24-SAfrica	3.54	1.17	1.76	1.72	14	10	14	10
25-RoAfrica	11.59	1.78	1.79	3.62	1	1	12	1

CONCLUSION

This document explains how we developed a reference scenario for a version of the PEP-w-t standard model with 25 regions and 18 commodities, on the basis of 1980-2050 historical data and projections produced by Fouré, Bénassy-Quéré and Fontagné (2012 – FBQF in the paper).

When we compare our reference scenario with FBQF’s projections, there are discrepancies, of course. But in spite of radical differences between FBQF’s aggregated model of national economies and the CGE approach of PEP-w-t, we have succeeded in aligning our reference scenario with FBQF’s very credible GDP and labor supply projections, without having to distort our model beyond recognition. Specifically, the calibration of $A_{z,t}^{VA}$, admittedly a sort of *deus ex machina*, yields values that may be reasonably interpreted as resulting from a plausible evolution of TFP.

Finally, we applied the growth accounting methodology to our reference scenario to confirm its consistency and to display its salient features.

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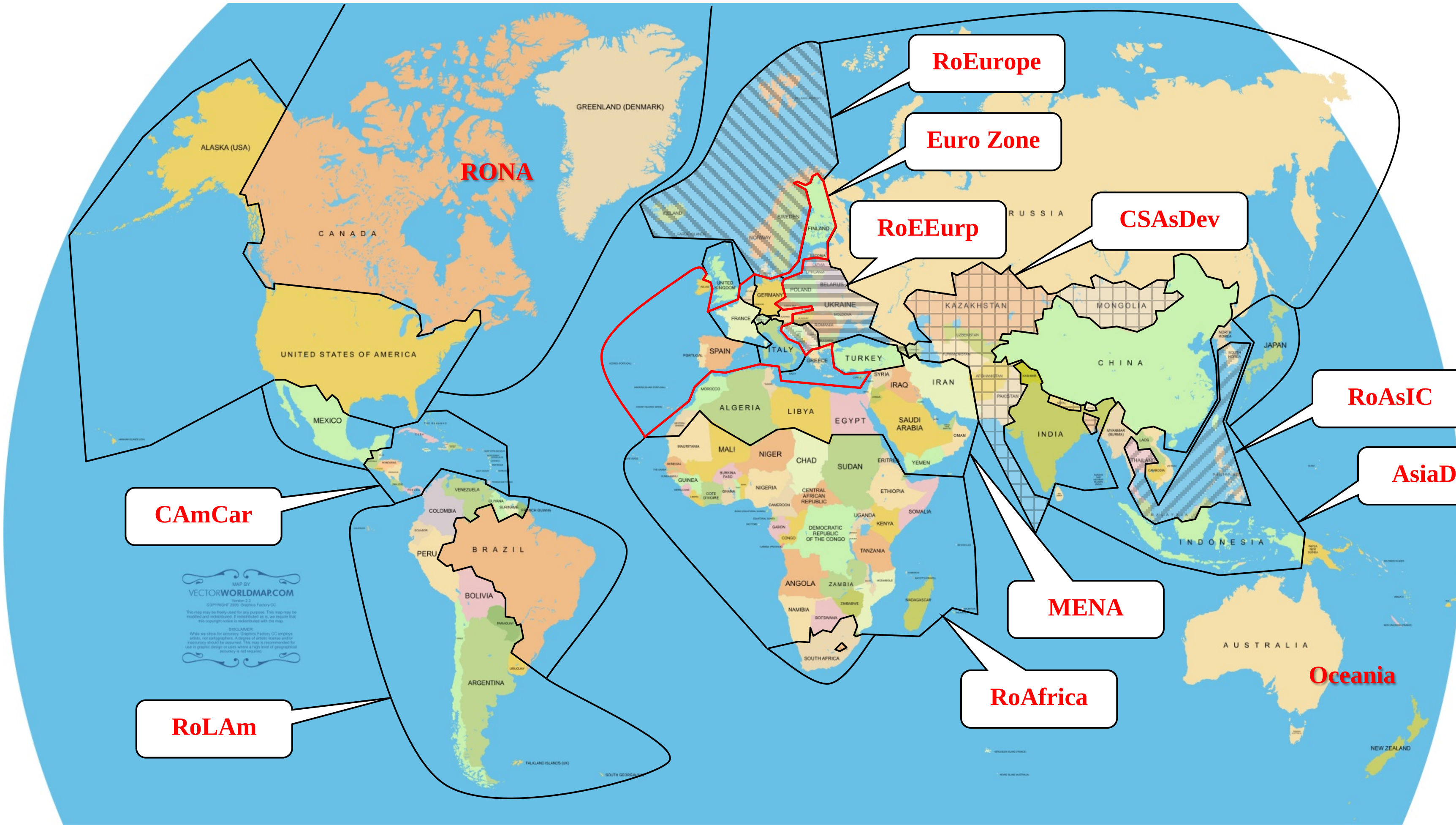
APPENDIX A – MAP OF THE PEP-w-t 25 REGIONS



Source : http://www.vectorworldmap.com/ and authors' drawing

Map legend: list of regions delineated :

PEP-25 regions		
01-Oceania	Oceania	
02-ChinaHK	China (incl. Hong Kong)	
03-Japan	Japan	
04-RoAsIC	Rest of Asian industrialized countries	
05-AsiaDev	Asia developing countries	
06-India	India	
07-CSAsDev	Central and South Asia developing	
08-USA	United States of America	
09-Mexico	Mexico	
10-RoNA	Rest of North America	
11-CAmCar	Central America and Caribbean	
12-Brazil	Brazil	
13-RoLAm	Rest of Latin America	
14-France	France	Euro Zone
15-Germany	Germany	Euro Zone
16-Italy	Italy	Euro Zone
18-REuroZ	Rest of Euro Zone	Euro Zone
17-UK	United Kingdom	
19-Russia	Russian Federation	
20-RoEEurp	Rest of Eastern Europe	
21-RoEurope	Rest of Europe	
22-Turkey	Turkey	
23-MENA	Middle-East and North Africa	
24-SAfrica	South Africa	
25-RoAfrica	Rest of Africa	

APPENDIX B – THE PEP-w-t 18 COMMODITIES

Table B1 - 18-commodity aggregation

PEP-18	Description
Agric	Agriculture
Mog	Minerals, oil and gaz
Food	Food products
Text	Textile and clothing
Wood	Wood and paper products
Petrochem	Petrochemical products
Metal	Metal products
TraEqp	Transportation equipment
Elctrn	Electronic equipment
MachEqp	Machinery and equipment nec
OthMan	Manufactures nec
ElGasWat	Electricity, gas and water distribution
Constr	Construction
Trade	Trade
Transport	Transport services
BusServ	Financial and business services
OthServ	Other services
Admin	Public Administration, Defense, Education, Health

APPENDIX C – COMPUTATION OF GROWTH FACTOR *exogro*

Let $y_{z,t}$ represent some variable that grows exogenously. If we were to postulate that it simply grows at the same rate as projected GDP, then we would have

$$y_{z,t+1} = \left(1 + g_{z,t}^{Y}\right) y_{z,t} \qquad \text{[C001]}$$

with

$$1 + g_{z,t}^{Y} = \tilde{Y}_{z,t+1} \Big/ \tilde{Y}_{z,t} \qquad \text{[C002]}$$

where

$g_{z,t}^{Y}$ is the past or projected growth rate of GDP in period *t* according to FBQF;

$\tilde{Y}_{z,t}$ is the past or projected GDP for period *t* according to FBQF.

Equation [C001] is tautologically equivalent to

$$y_{z,t+1} = \left(1 + g_{z,t}^{LS}\right) \left(\frac{1 + g_{z,t}^{Y}}{1 + g_{z,t}^{LS}} \right) y_{z,t} \qquad \text{[C003]}$$

with

$$1 + g_{z,t}^{LS} = \tilde{L}_{z,t+1} \Big/ \tilde{L}_{z,t} \qquad \text{[C004]}$$

where

$g_{z,t}^{LS}$ is the past or projected growth rate of the economically active population in period *t* according to FBQF;

$\tilde{L}_{z,t}$ is the past or projected economically active population for period *t* according to FBQF.

The growth rate of the projected output per worker (labor productivity) is given by

$$\left(\frac{\tilde{Y}_{z,t+1} \big/ \tilde{L}_{z,t+1}}{\tilde{Y}_{z,t} \big/ \tilde{L}_{z,t}} \right) - 1 = \left(\frac{\tilde{Y}_{z,t+1} \big/ \tilde{Y}_{z,t}}{\tilde{L}_{z,t+1} \big/ \tilde{L}_{z,t}} \right) - 1 = \left(\frac{1 + g_{z,t}^{Y}}{1 + g_{z,t}^{LS}} \right) - 1 \qquad \text{[C005]}$$

Now, as a matter of fact, in the projections of FBQF, the growth rate of labor, based on demographic variables, is generally much more stable than the growth rate of either GDP, or output per worker. In order to obtain a smooth reference scenario, we replace the ratio $\left(\frac{1 + g_{z,t}^{Y}}{1 + g_{z,t}^{LS}} \right)$ in equation [C003] with its ten-year moving average, $\overline{\left(\frac{1+g_{z,t}^{Y}}{1+g_{z,t}^{LS}}\right)}$, defined as

$$\overline{\left(\frac{1+g_{t}^{Y}}{1+g_{t}^{LS}}\right)} = \frac{1}{10}\sum_{\theta=t-5}^{t+4}\left(\frac{1+g_{\theta}^{Y}}{1+g_{\theta}^{LS}}\right), \text{ for } t = 2007,...,2050 \qquad \text{[C006]}$$

where, for $t > 2049$,

$$g_{t}^{Y} = g_{2049}^{Y} \text{ and } g_{t}^{LS} = g_{2049}^{LS} \text{ for } t > 2049 \qquad \text{[C007]}$$

Then we have

$$y_{z,t+1} = \left(1+g_{z,t}^{LS}\right)\overline{\left(\frac{1+g_{z,t}^{Y}}{1+g_{z,t}^{LS}}\right)}y_{t} \qquad \text{[C008]}$$

We set *exogro* at 1 in the base year, and then compute

$$exogro_{z,t+1} = \left(1+g_{z,t}^{LS}\right)\overline{\left(\frac{1+g_{z,t}^{Y}}{1+g_{z,t}^{LS}}\right)}exogro_{z,t} \qquad \text{[C009]}$$

where $exogro_{z,t}$ is our exogenous growth factor. Then, the exogenous values of $y_{z,t}$ are given by

$$y_{z,t} = exogro_{z,t}\,y_{z,'2007'} \qquad \text{[C010]}$$

where 2007 is the model’s base year.

APPENDIX D – COMPUTATION OF THE INDEX OF REAL CURRENT ACCOUNT IN TERMS OF THE INTERNATIONAL CURRENCY

The first step is to compute what $CABX_{z,t}$ would be if it were to grow according to $exogro_{z,t}$. We define two variables, one for positive CABs (SURplus) and another for negative ones (DEFicit) :

$$CABX_{z,t}^{SUR} = \begin{cases} CABX_z^O \ exogro_{z,t} \text{ if } CABX_z^O > 0 \\ 0 \text{ otherwise} \end{cases} \quad [D001]$$

$$CABX_{z,t}^{DEF} = \begin{cases} CABX_z^O \ exogro_{z,t} \text{ if } CABX_z^O < 0 \\ 0 \text{ otherwise} \end{cases} \quad [D002]$$

Then we compute what the world sum of current account balances Bal_t would be with these values:

$$Bal_t = \sum_z CABX_{z,t}^{SUR} + \sum_z CABX_{z,t}^{DEF} \quad [D003]$$

Bal_t is the amount of disequilibrium to be eliminated, or, in other words, the burden of adjustment. Define

$$\mu_t = \frac{\sum_z CABX_{z,t}^{SUR}}{\sum_z CABX_{z,t}^{SUR} + \sum_z \left| CABX_{z,t}^{DEF} \right|} \quad [D004]$$

where

$\left| CABX_{z,t}^{DEF} \right|$ is the absolute value of $CABX_{z,t}^{DEF}$,

and obtain the revised values $CABX_{z,t}^{SUR2}$ and $CABX_{z,t}^{DEF2}$ as

$$CABX_{z,t}^{SUR2} = CABX_{z,t}^{SUR} - \left[\frac{CABX_{z,t}^{SUR}}{\sum_{zj} CABX_{zj,t}^{SUR}} \right] \mu_t Bal_t \quad [D005]$$

$$CABX_{z,t}^{DEF2} = CABX_{z,t}^{DEF} - \left[\frac{CABX_{z,t}^{DEF}}{\sum_{zj} CABX_{zj,t}^{DEF}} \right] (1 - \mu_t) Bal_t \quad [D006]$$

Then the adjustment on every current account balance initially projected using $exogro_{z,t}$ is proportional to its absolute size, whether it be a surplus or a deficit.

Equations [D005] and [D006] may be written equivalently as

$$CABX_{z,t}^{SUR2} = CABX_{z,t}^{SUR}\left[1-\frac{\mu_t Bal_t}{\sum_{zj} CABX_{zj,t}^{SUR}}\right] \qquad \text{[D007]}$$

$$CABX_{z,t}^{DEF2} = CABX_{z,t}^{DEF}\left[1-\frac{(1-\mu_t)Bal_t}{\sum_{zj} CABX_{zj,t}^{DEF}}\right] \qquad \text{[D008]}$$

It is easily verified that

$$\begin{aligned}\sum_z CABX_{z,t}^{SUR2} + \sum_z CABX_{z,t}^{DEF2} &= \sum_z \left\{CABX_{z,t}^{SUR} - \left[\frac{CABX_{z,t}^{SUR}}{\sum_{zj} CABX_{zj,t}^{SUR}}\right]\mu_t Bal_t\right\} \\ &+ \sum_z \left\{CABX_{z,t}^{DEF} - \left[\frac{CABX_{z,t}^{DEF}}{\sum_{zj} CABX_{zj,t}^{DEF}}\right](1-\mu_t)Bal_t\right\}\end{aligned} \qquad \text{[D009]}$$

$$\begin{aligned}\sum_z CABX_{z,t}^{SUR2} + \sum_z CABX_{z,t}^{DEF2} &= \sum_z CABX_{z,t}^{SUR} - \mu_t Bal_t \sum_z \left[\frac{CABX_{z,t}^{SUR}}{\sum_{zj} CABX_{zj,t}^{SUR}}\right] \\ &+ \sum_z CABX_{z,t}^{DEF} - (1-\mu_t)Bal_t \sum_z \left[\frac{CABX_{z,t}^{DEF}}{\sum_{zj} CABX_{zj,t}^{DEF}}\right]\end{aligned} \qquad \text{[D010]}$$

$$\sum_z CABX_{z,t}^{SUR2} + \sum_z CABX_{z,t}^{DEF2} = \sum_z CABX_{z,t}^{SUR} - \mu_t Bal_t + \sum_z CABX_{z,t}^{DEF} - (1-\mu_t)Bal_t \qquad \text{[D011]}$$

$$\sum_z CABX_{z,t}^{SUR2} + \sum_z CABX_{z,t}^{DEF2} = \sum_z CABX_{z,t}^{SUR} + \sum_z CABX_{z,t}^{DEF} - Bal_t \qquad \text{[D012]}$$

$$\sum_z CABX_{z,t}^{SUR2} + \sum_z CABX_{z,t}^{DEF2} = Bal_t - Bal_t = 0 \qquad \text{[D013]}$$

Then compute

$$cabix_{z,t} = \frac{CABX_{z,t}^{SUR2} + CABX_{z,t}^{DEF2}}{CABX_z^{O}} \qquad \text{[D014]}$$

This is equivalent to

$$cabix_{z,t} = \begin{cases} \dfrac{CABX_{z,t}^{SUR2}}{CABX_z^O} \text{ if } CABX_z^O > 0 \\ \\ \dfrac{CABX_{z,t}^{DEF2}}{CABX_z^O} \text{ if } CABX_z^O < 0 \end{cases} \quad \text{[D015]}$$

Finally, the exogenously determined current account balances will be defined as

$$CABX_{z,t} = cabix_{z,t} CABX_z^O \quad \text{[015]}$$

APPENDIX E – GROWTH DECOMPOSITION OF THE PEP-w-t REFERENCE SCENARIO

Solow (1957) laid the foundations of the measurement of productivity change using "growth accounting". An abundant literature followed, in which Domar (1961), Jorgenson and Griliches (1969) and Hulten (1978) stand out. According to that approach, measuring changes in productivity essentially consists in decomposing growth into its contributing factors. First, one must measure the growth of production, to then relate it to the growth of inputs in order to assign each of the inputs taken into account a fraction of the growth of production; finally, the residual (the fraction of growth that has not been assigned to any input) is interpreted as an effect of total (or multi-) factor productivity growth.

The presentation here takes its inspiration from Baldwin *et al*. (2007).

E1. Theory of growth decomposition

E1.1 Growth decomposition in continuous time

The general principle of growth decomposition can be represented by the following equation:

$$T\dot{F}P = \dot{Q} - \sum_i \frac{P_i X_i}{PQ} \dot{X}_i = \dot{Q} - \sum_i s_i \dot{X}_i \qquad \text{[E001]}$$

where

$T\dot{F}P$ is the growth rate of TFP

$\dot{Q}$ is the growth rate of the quantity of production : $\dot{Q} = \frac{d \ln Q}{dt} = \frac{1}{Q}\frac{dQ}{dt}$;

P is the price of the product;

$\dot{X}_i$ is the rate of increase of the quantity of input *i*: $\dot{X}_i = \frac{d \ln X_i}{dt} = \frac{1}{X_i}\frac{dX_i}{dt}$;

P_i is the price of input *i*;

s_i is the share of input *i* in the product's value: $s_i = \frac{P_i X_i}{\sum_j P_j X_j} = \frac{P_i X_i}{PQ}$

E1.2 Discrete time approximation

Since economic data are not collected in continuous time, practical calculations resort to a discrete time approximation, making the following substitutions:

$\Delta \ln Q_t = \ln Q_t - \ln Q_{t-1} = \ln \frac{Q_t}{Q_{t-1}}$ is substituted for $\dot{Q} = \frac{d \ln Q}{dt} = \frac{1}{Q}\frac{dQ}{dt}$;

$$\Delta \ln X_{i,t} = \ln X_{i,t} - \ln X_{i,t-1} = \ln \frac{X_{i,t}}{X_{i,t-1}} \text{ is substituted for } \dot{X} = \frac{d \ln X_i}{dt} = \frac{1}{X_i}\frac{dX_i}{dt};$$

$$\Delta \ln TFP_t = \ln TFP_t - \ln TFP_{t-1} = \ln \frac{TFP_t}{TFP_{t-1}} \text{ is substituted for } T\dot{F}P = \frac{1}{Q}\frac{dF}{dt} = \frac{F}{Q}\frac{1}{F}\frac{dF}{dt} = \frac{F}{Q}\frac{d \ln F}{dt};$$

$\bar{s}_{i,t} = \left(s_{i,t} + s_{i,t-1}\right)/2$ for s_i.

It follows that

$$\Delta \ln TFP_t = \Delta \ln Q_t - \sum_i \bar{s}_{i,t} \Delta \ln X_{i,t} \qquad \text{[E002]}$$

Using the properties of logarithms

$$\ln z^v = v \ln z$$

and

$$\ln(xy) = \ln x + \ln y,$$

we can write

$$\Delta \ln TFP_t = \ln\left(Q_t / Q_{t-1}\right) - \ln \prod_i \left[\left(X_{it} / X_{i,t-1}\right)^{\bar{s}_{it}}\right] \qquad \text{[E003]}$$

where the second term on the right-hand side is the Törnqvist quantity index.

Once $\Delta \ln TFP_t$ has been computed with formula E002, the rate of growth of TFP is computed using

$$e^{\Delta \ln TFP_t} - 1 \equiv e^{\ln\left(TFP_t / TFP_{t-1}\right)} - 1 \equiv \frac{TFP_t}{TFP_{t-1}} - 1 \qquad \text{[E004]}$$

E2. Application to the PEP-w-t reference scenario

E2.1 Industry value-added growth decomposition

The growth decomposition may be applied to PEP-w-t one industry at a time.

Factor shares are calculated using the model equation for the price of value added:

$$PVA_{j,z,t} = \frac{WC_{j,z,t} LDC_{j,z,t} + RC_{j,z,t} KDC_{j,z,t}}{VA_{j,z,t}} \qquad \text{[E005]}$$

where

$KDC_{j,z,t}$: Demand for composite capital by industry *j* in region *z* in period *t*

$LDC_{j,z,t}$: Demand for composite labor by industry *j* in region *z* in period *t*

$PVA_{j,z,t}$: Price of industry *j* value added in region *z* in period *t*

$RC_{j,z,t}$: Rental rate of industry *j* composite capital in region *z* in period *t*

$VA_{j,z,t}$: Value added of industry *j* in region *z* in period *t*

$WC_{j,z,t}$: Wage rate of industry *j* composite labor in region *z* in period *t*

This yields

$$s_{LDC,j,z,t} = \frac{WC_{j,z,t} LDC_{j,z,t}}{PVA_{j,z,t} VA_{j,z,t}} \qquad [E006]$$

$$s_{KDC,j,z,t} = \frac{RC_{j,z,t} KDC_{j,z,t}}{PVA_{j,z,t} VA_{j,z,t}} = 1 - s_{LDC,j,z,t} \qquad [E007]$$

and

$$\bar{s}_{LDC,j,z,t} = \frac{s_{LDC,j,z,t} + s_{LDC,j,z,t-1}}{2} \qquad [E008]$$

$$\bar{s}_{KDC,j,z,t} = \frac{s_{KDC,j,z,t} + s_{KDC,j,z,t-1}}{2} \qquad [E009]$$

Calculations also require

$$\Delta \ln VA_{j,z,t} = \ln VA_{j,z,t} - \ln VA_{j,z,t-1} = \ln\left(VA_{j,z,t} / VA_{j,z,t-1}\right) \qquad [E010]$$

$$\Delta \ln LDC_{j,z,t} = \ln LDC_{j,z,t} - \ln LDC_{j,z,t-1} = \ln\left(LDC_{j,z,t} / LDC_{j,z,t-1}\right) \qquad [E011]$$

$$\Delta \ln KDC_{j,z,t} = \ln KDC_{j,z,t} - \ln KDC_{j,z,t-1} = \ln\left(KDC_{j,z,t} / KDC_{j,z,t-1}\right) \qquad [E012]$$

The industry growth residual $TFP^{Ind}_{j,z,t}$, which may be interpreted as the change in total factor productivity in the industry, is computed from

$$\Delta \ln TFP^{Ind}_{j,z,t} = \Delta \ln VA_{j,z,t} - \bar{s}_{LDC,j,z,t} \Delta \ln LDC_{j,z,t} - \bar{s}_{KDC,j,z,t} \Delta \ln KDC_{j,z,t} \qquad [E013]$$

E2.2 Economy-wide GDP growth decomposition

To perform economy-wide growth accounting, first combine the two following model equations

$$TIPT_{z,t} = \sum_j TIP_{j,z,t} \qquad [E014]$$

$$GDP^{BP}_{z,t} = \sum_j PVA_{j,z,t} VA_{j,z,t} + TIPT_{z,t} \qquad [E015]$$

where

$GDP_{z,t}^{BP}$: GDP at basic prices in region z in period t

$TIP_{j,z,t}$: Government revenue from taxes on industry j production in region z (excluding taxes directly related to the use of capital and labor) in period t

$TIPT_{z,t}$: Total government revenue from production taxes in region z (excluding taxes directly related to the use of capital and labor) in period t

and write

$$GDP_{z,t}^{BP} = \sum_j PVA_{j,z,t} VA_{j,z,t} + \sum_j TIP_{j,z,t} \qquad \text{[E016]}$$

Let

$$\tau_{j,z,t} = \frac{TIP_{j,z,t}}{PVA_{j,z,t} VA_{j,z,t}} \qquad \text{[E017]}$$

and write E016 as

$$GDP_{z,t}^{BP} = \sum_j PVA_{j,z,t} VA_{j,z,t} + \sum_j \tau_{j,z,t} PVA_{j,z,t} VA_{j,z,t} \qquad \text{[E018]}$$

$$GDP_{z,t}^{BP} = \sum_j \left(1 + \tau_{j,z,t}\right) PVA_{j,z,t} VA_{j,z,t} \qquad \text{[E019]}$$

Combine [E019] and model equation [E005]

$$PVA_{j,z,t} = \frac{WC_{j,z,t}\, LDC_{j,z,t} + RC_{j,z,t}\, KDC_{j,z,t}}{VA_{j,z,t}} \qquad \text{[E005]}$$

to obtain

$$GDP_{z,t}^{BP} = \sum_j \left(1 + \tau_{j,z,t}\right)\left(WC_{j,z,t}\, LDC_{j,z,t} + RC_{j,z,t}\, KDC_{j,z,t}\right) \qquad \text{[E020]}$$

$$GDP_{z,t}^{BP} = \sum_j \left(1 + \tau_{j,z,t}\right) WC_{j,z,t}\, LDC_{j,z,t} + \sum_j \left(1 + \tau_{j,z,t}\right) RC_{j,z,t}\, KDC_{j,z,t} \qquad \text{[E021]}$$

Interpreting $\left(1 + \tau_{j,z,t}\right) WC_{j,z,t}$ as the tax-inclusive price of composite labor, and $\left(1 + \tau_{j,z,t}\right) RC_{j,z,t}$ as the tax-inclusive price of composite capital, define the Fisher quantity indexes of aggregate labor and capital as:

$$QIXLDC_{z,t} = \sqrt{\frac{\sum_j \left(1+\tau^O_{j,z}\right) WC^O_{j,z}\ LDC_{j,z,t}}{\sum_j \left(1+\tau^O_{j,z}\right) WC^O_{j,z}\ LDC^O_{j,z}} \frac{\sum_j \left(1+\tau_{j,z,t}\right) WC_{j,z,t}\ LDC_{j,z,t}}{\sum_j \left(1+\tau_{j,z,t}\right) WC_{j,z,t} LDC^O_{j,z}}} \quad \text{[E022]}$$

$$QIXKDC_{z,t} = \sqrt{\frac{\sum_j \left(1+\tau^O_{j,z}\right) RC^O_{j,z}\ KDC_{j,z,t}}{\sum_j \left(1+\tau^O_{j,z}\right) RC^O_{j,z}\ KDC^O_{j,z}} \frac{\sum_j \left(1+\tau_{j,z,t}\right) RC_{j,z,t}\ KDC_{j,z,t}}{\sum_j \left(1+\tau_{j,z,t}\right) RC_{j,z,t} KDC^O_{j,z}}} \quad \text{[E023]}$$

The corresponding price indexes are

$$PIXLDC_{z,t} = \sqrt{\frac{\sum_j \left(1+\tau_{j,z,t}\right) WC_{j,z,t}\ LDC^O_{j,z}}{\sum_j \left(1+\tau^O_{j,z}\right) WC^O_{j,z}\ LDC^O_{j,z}} \frac{\sum_j \left(1+\tau_{j,z,t}\right) WC_{j,z,t}\ LDC_{j,z,t}}{\sum_j \left(1+\tau^O_{j,z}\right) WC^O_{j,z} LDC_{j,z,t}}} \quad \text{[E024]}$$

$$PIXKDC_{z,t} = \sqrt{\frac{\sum_j \left(1+\tau_{j,z,t}\right) RC_{j,z,t}\ KDC^O_{j,z}}{\sum_j \left(1+\tau^O_{j,z}\right) RC^O_{j,z}\ KDC^O_{j,z}} \frac{\sum_j \left(1+\tau_{j,z,t}\right) RC_{j,z,t}\ KDC_{j,z,t}}{\sum_j \left(1+\tau^O_{j,z}\right) RC^O_{j,z} KDC_{j,z,t}}} \quad \text{[E025]}$$

with the following identities

$$\frac{\sum_j \left(1+\tau_{j,z,t}\right) WC_{j,z,t}\ LDC_{j,z,t}}{\sum_j \left(1+\tau^O_{j,z}\right) WC^O_{j,z}\ LDC^O_{j,z}} = PIXLDC_{z,t} QIXLDC_{z,t} \quad \text{[E026]}$$

$$\frac{\sum_j \left(1+\tau_{j,z,t}\right) RC_{j,z,t}\ KDC_{j,z,t}}{\sum_j \left(1+\tau^O_{j,z}\right) RC^O_{j,z}\ KDC^O_{j,z}} = PIXKDC_{z,t} QIXKDC_{z,t} \quad \text{[E027]}$$

Aggregate factor shares are given by

$$s_{LDC,z,t} = \frac{\sum_j \left(1+\tau_{j,z,t}\right) WC_{j,z,t}\ LDC_{j,z,t}}{GDP^{BP}_{z,t}} \quad \text{[E028]}$$

$$s_{KDC,z,t} = \frac{\sum_j \left(1+\tau_{j,z,t}\right) RC_{j,z,t}\ KDC_{j,z,t}}{GDP^{BP}_{z,t}} \quad \text{[E029]}$$

and we have

$$\bar{s}_{LDC,z,t} = \frac{s_{LDC,z,t} + s_{LDC,z,t-1}}{2} \quad \text{[E030]}$$

$$\bar{s}_{KDC,z,t} = \frac{s_{KDC,z,t} + s_{KDC,z,t-1}}{2} \quad \text{[E031]}$$

The economy-wide growth residual $TFP_{z,t}$, which may be interpreted as the change in total factor productivity, is computed from

$$\Delta \ln TFP_{j,zt} = \Delta \ln GDP_{z,t}^{BP\_REAL} - \bar{s}_{LDC,z,t} \Delta \ln QIXLDC_{z,t} - \bar{s}_{KDC,j,z,t} \Delta \ln QIXKDC_{z,t} \quad \text{[E032]}$$

All these elements are computed in a GAMS program. They are then used in the same program to compute the real GDP that would have been produced without productivity growth.

$$\frac{GDP_{z,t}^{NoTFP}}{GDP_{z,t-1}^{NoTFP}} = \exp\left[\bar{s}_{LDC,z,t} \Delta \ln QIXLDC_{z,t} + \bar{s}_{KDC,j,z,t} \Delta \ln QIXKDC_{z,t} \right] \quad \text{[E033]}$$

$$\frac{GDP_{z,t}^{NoTFP}}{GDP_{z,t-1}^{NoTFP}} = \exp\left[\bar{s}_{LDC,z,t} \Delta \ln QIXLDC_{z,t} \right] \exp\left[\bar{s}_{KDC,j,z,t} \Delta \ln QIXKDC_{z,t} \right] \quad \text{[E034]}$$

E2.3 Growth accounting

Substitute equation [E034] into the following identity:

$$\frac{GDP_{z,t}^{NoTFP}}{GDP_{z,t1}^{NoTFP}} = \prod_{\theta=t1+1}^{t} \frac{GDP_{z,\theta}^{NoTFP}}{GDP_{z,\theta-1}^{NoTFP}} \quad \text{[E035]}$$

where $t1$ is the first model period (base year), to obtain

$$\frac{GDP_{z,t}^{NoTFP}}{GDP_{z,t1}^{NoTFP}} = \left(\prod_{\theta=t1+1}^{t} \exp\left[\bar{s}_{LDC,z,\theta} \Delta \ln QIXLDC_{z,\theta}\right] \right) \left(\prod_{\theta=t1+1}^{t} \exp\left[\bar{s}_{KDC,z,\theta} \Delta \ln QIXKDC_{z,\theta}\right] \right) \quad \text{[E036]}$$

Now develop [020]:

$$GDP_{z,t}^{BP\_REAL} = A_{z,t}^{VA} \, GDP_{z,t}^{NoTFP} \quad \text{[020]}$$

$$\frac{GDP_{z,t}^{BP\_REAL}}{GDP_{z,t1}^{BP\_REAL}} = \frac{GDP_{z,t}^{BP\_REAL}}{GDP_{z}^{BP\_REALO}} = \frac{A_{z,t}^{VA} \, GDP_{z,t}^{NoTFP}}{A_{z}^{VAO} \, GDP_{z,t1}^{NoTFP}} \quad \text{[E037]}$$

Substitute E036 into E037:

$$\frac{GDP_{z,t}^{BP\_REAL}}{GDP_{z}^{BP\_REALO}} = \frac{A_{z,t}^{VA}}{A_{z}^{VAO}} \left(\prod_{\theta=t1+1}^{t} \exp\left[\bar{s}_{LDC,z,\theta} \Delta \ln QIXLDC_{z,\theta}\right]\right) \left(\prod_{\theta=t1+1}^{t} \exp\left[\bar{s}_{KDC,z,\theta} \Delta \ln QIXKDC_{z,\theta}\right]\right) \quad [E038]$$

$$\ln\left(\frac{GDP_{z,t}^{BP\_REAL}}{GDP_{z}^{BP\_REALO}}\right) = \ln\left(\frac{A_{z,t}^{VA}}{A_{z}^{VAO}}\right) + \ln\left(\prod_{\theta=t1+1}^{t} \exp\left[\bar{s}_{LDC,z,\theta} \Delta \ln QIXLDC_{z,\theta}\right]\right) + \ln\left(\prod_{\theta=t1+1}^{t} \exp\left[\bar{s}_{KDC,z,\theta} \Delta \ln QIXKDC_{z,\theta}\right]\right) \quad [E039]$$

$$\ln\left(\frac{GDP_{z,t}^{BP\_REAL}}{GDP_{z}^{BP\_REALO}}\right) = \ln\left(\frac{A_{z,t}^{VA}}{A_{z}^{VAO}}\right) + \sum_{\theta=t1+1}^{t} \left[\bar{s}_{LDC,z,\theta} \Delta \ln QIXLDC_{z,\theta}\right] + \sum_{\theta=t1+1}^{t} \left[\bar{s}_{KDC,z,\theta} \Delta \ln QIXKDC_{z,\theta}\right] \quad [E040]$$

$$\frac{GDP_{z,t}^{BP\_REAL}}{GDP_{z}^{BP\_REALO}} = \frac{A_{z,t}^{VA}}{A_{z}^{VAO}} \exp\left\{ \sum_{\theta=t1+1}^{t} \left[\bar{s}_{LDC,z,\theta} \Delta \ln QIXLDC_{z,\theta}\right]\right\} \exp\left\{ \sum_{\theta=t1+1}^{t} \left[\bar{s}_{KDC,z,\theta} \Delta \ln QIXKDC_{z,\theta}\right]\right\} \quad [021]$$

This is the growth accounting formula.