Oil, metals and minerals : World prices and Quebec's regions (Results from a CGE model)

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partnership for economic policy pep

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SUMMARY

The authors have developed a computable general equilibrium (CGE) model of the province of Quebec, based on a new, relatively detailed social accounting matrix (SAM) for 2011. From the whole-Quebec SAM, they elaborated a multiregional SAM using, in particular, regional data produced by Institut de la statistique du Québec (ISQ), most notably estimates of regional GDP by industry. The two SAMs stand out for the care that was taken in conceiving their structure, and then in quantifying its elements from publicly available data, completed with information from various sources, proportionality assumptions and bi-proportional adjustments (RAS technique), subject to accounting identities and benchmark data.

Quebec is broken down into 16 analytical regions: the Montreal census metropolitan area (CMA), subdivided in three (Montreal, Laval, Rest of the CMA); the 5 other CMAs in Quebec; 6 other analytical regions, called "peri-metropolitan", non-metropolitan parts of the administrative regions (AR) having territory in common with each CMA; finally, two peripheral regions, *Rest-of-the-North* and *East*. Interregional trade flows were generated from regional output–domestic demand balances using a gravity model.

The multiregional model built on the basis of that SAM, MEGBEC, is unique in Quebec. It is a recursive dynamic model inspired from PEP-1-t, considerably modified to take full advantage of the wealth of available data and adapt it to the specific structure of the Quebec economy. The MEGBEC model is used here to simulate the impact on Quebec regions of the drop in world prices of oil and metals and minerals. Results show that the fall in oil prices has a positive but diffuse effect on the Quebec economy, while the drop in the prices of metals and minerals has a negative impact and hits regions unequally: regions where mining and primary metal manufacturing are concentrated are hardest hit. Combining both shocks had a slightly positive overall impact, but a negative one in regions dependent on the prices of metals and minerals.

OIL, METALS AND MINERALS : WORLD PRICES AND QUEBEC'S REGIONS (RESULTS FROM A CGE MODEL)

1. Introduction

The multiregional computable general equilibrium (CGE) model MEGBEC is unique: to our knowledge, there is no other multiregional CGE model of Quebec; there is not even a multiretional input-output model. The model distinguishes 16 so-called "analytical regions" (ANAR). These regions were delineated in Lemelin (2013)[1] to define geographical entities that are economically meaningful and cover the whole territory, while taking into account available economic data sources. Now, economic data published by Institut de la statistique du Québec relate either to census metropolitan areas (CMAs) – and then all the rest of the province is aggregated as "non-metropolitan" –, or to "administrative regions" (ARs) – not suitable for economic analysis. By combining data relating to CMAs and ARs using addition and subtraction, it is possible to define regions in such a way that the organic character, so to speak, of CMAs is maintained, but non-metropolitan areas are not lumped together. Appendix 3 provides a map and a definition of the analytical regions. Each of the six CMAs is an analytical region, except the Montreal CMA, which is subdivided in three, a refinement made possible by the fact that the ARs of Montréal (Montreal island) and Laval are completely embedded in the CMA. Six other analytical regions, which we call "peri-metropolitan" consist of the non-metropolitan parts of the ARs having territory in common with the CMA. Finally, we have defined two peripheral regions, *Rest-of-the-North* and *East*. Admittedly, this geographical breakdown is not ideal for economic analysis. But it is a realistic compromise which, as we intend to show, makes it possible to obtain enlightening results.

The MEGBEC model is based upon a rather detailed social accounting matrix (SAM), with 44 industries, 64 products, 4 production factors and 20 economic agent accounts (Appendix 2).[2] The version of the model which we present here is operational, but somewhat like a freshly launched ocean liner, it floats and can sail, but not all of its superstructure is in place.

The issue we want to examine is related to recent economic developments, in particular, the fall in the prices of oil, and of metals and minerals. Oil-producing provinces in Canada have been hard hit by the crude oil price collapse, but the same may have benefitted oil-importing provinces such as Quebec. On

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1 It is likely that others have used the same geographical breakdown.

2 To our knowledge, there are only two other CGE models of Quebec: the Quebec Ministry of Finance model, not open to use by researchers, and a model developed by the *Groupe de Recherche en Économie et Développement International* (GREDI) of the Université de Sherbrooke. See the corresponding section regarding other Quebec CGE models in the list of references.

the other hand, the euphoria of the raw materials boom has given way to a slump which has left behind the Plan Nord of the Quebec government. Of the two shocks, one positive, the other negative, which have hit the Quebec economy, we try to see which one dominates, and how the impact varies from region to region.

The rest of this paper is organized as follows. In Section 2, we describe how the SAM has been elaborated, a task which was accomplished over a fifteen-month period, with very modest means. Section 3 presents the model, evolved from the PEP-1-t model, but nonetheless markedly different. In Section 4 the basic scenario and the simulation scenarios are defined, and the results are analysed. Concluding remarks complete the article.

2. Elaboration of the social accounting matrix

The SAM underlying the model was elaborated in two phases. We first built a SAM for Quebec as a whole, the general structure of which is illustrated in Appendix 4 using an aggregated table. The SAM represents transactions flows in the economy following the format put forward in the United Nations national accounting system (INTER-SECRETARIAT WORKING GROUP ON NATIONAL ACCOUNTS, 2009)[3].

Once completed, the SAM of Quebec as a whole was treated as a benchmark for the regional SAMs. The latter were developed using a combination of regional data and regional allocators to distribute SAM values among the regions.

We now proceed to present a brief account of the very involved process of SAM construction.

2.1 ELABORATION OF THE SAM FOR QUEBEC AS A WHOLE

The SAM for Quebec as a whole was elaborated from the input-output (IO) tables[4] for Quebec in 2011, produced by the Industry Acounts Division of Statistics Canada. The provincial IO tables have confidential entries, which have been reconstituted using corresponding data for Canada and applying the minimum cross-entropy method (RAS technique).

Moreover, the publicly available IO tables are at the S-aggregation level, where all manufacturing is lumped together. Using various information sources, proportionality assumptions, and bi-proportional adjustments (RAS technique), it was possible to disaggregate manufacturing into 19 industries. The main source of information was CANSIM Table 381-0031, which presents provincial gross output by sector

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3 Regarding the general structure of the SAM and the national accounting concepts, the interested reader is referred to Decaluwé *et al*. (2013b).

4 Statistics Canada now calls these tables “Supply and use tables”.

and industry. We have also used results from the Quebec input-output model (*Modèle intersectoriel du Québec* – MISQ), which were available on the Institut de la statistique du Québec website for awhile.

At this stage, the tables are at basic prices, that is, without taxes on products or transport and trade margins. The balanced final and intermediate demand tables were revalued at market prices ("purchasers' prices" in Statistics Canada terminology), using corresponding Statistics Canada tables; and the SAM margin accounts were created. Finally, the SAM was adjusted to be exactly in line with the Quebec income and expenditure accounts (*Comptes économiques des revenus et dépenses du Québec*) published by the Institut de la statistique du Québec on the basis of Statistics Canada data.

From that initial SAM, several improvements have been applied.

First, given that mixed income (also called "Net income of unincorporated business") includes factor payments to capital and to labor, we have separated the two by subtracting the employment income of self-employed workers, according to CANSIM Table 383-0031. This will make it possible, when using the model, to take into account, at least partly, the expanding role of self-employment in the labor market.

Secondly, we have estimated property income paid and received by economic agents (interest, dividends, etc., all treated as transfers in the national accounts), using available data for Quebec and completing from data relating to Canada, with proportionality assumptions. For want of information regarding property income cross-flows between Quebec and the rest of Canada (RoC), we made the hypothesis that Canada is financially fully integrated, without friction; that seems to be less restrictive than assuming no cross-flows. So we imputed cross-flows between Quebec and the RoC applying bi-proportional adjustment to property income flows, subject to incomes paid and received and estimated flows relating to the rest of the world (RoW). Such "audacity" is justified by the fact that under standard modeling, property income received from the RoC would be exogenous and fixed, while that paid to the RoC would be proportional to the payer's income, or at least would be endogenous. If, as we believe, financial income cross-flows between Quebec and the RoC are large, ignoring them in simulations would grossly under-estimate changes in the amounts of property income paid to the RoC by Quebec's economy.

Thirdly, to take advantage of the fact that capital stock and investment expenditures are detailed by asset category, on the basis of CANSIM Table 031-0005, the rate of capital depreciation in each industry is a weighted average of the depreciation rates of the assets that make it up.[5]

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5 We had hoped to better take into account the different depreciation rates. Alas, the composition of investments and that of the capital stock according to Table 031-0005 are different from one another, and we could not reconcile them using an investment model that would have taken account the various speeds at which assets depreciate. For that, the composition of the capital stock would have to evolve from period to period, and we didn't have enough time to develop the corresponding model.

Lastly, once we had estimated the capital stock and investment expenditures of public sector industries (Appendix 2, industries 40 to 44) by asset category, we extracted stocks of, and investment in road infrastructures.[6] Introducing that distinction makes it possible, in particular, to reproduce observed public investment expenditure patterns without adding the infrastructure capital to the productive capital directly related to the activity of public sector industries. Eventually, it will be possible to conduct studies of the impact of infrastructures along the lines of Bahan *et al*. (2011) and Boccanfuso *et al*. (2014a and b).

2.2 ELABORATION OF THE MULTIREGIONAL SAM

The multiregional SAM consists of 16 SAMs in a diagonal, to which are added 23 supraregional acounts, as shown in Figure 1. Each one of the 16 SAMs has the same structure as the whole-Quebec SAM illustrated in Appendix 4, except that some of the agent accounts and the savings-investment accounts are missing. Accounts *ISBL* (Non-profit institutions serving households – NPISH) and *RPROPRI* (Property income – interest payments, dividends, etc.) are supraregional accounts. In addition, accounts *FED* (Federal government), *RPC* (Canada Pension Plan), *PROV* (Provincial government) and *RRQ* (*Régime de rentes du Québec*) have a supraregional counterpart where regional surpluses and deficits, together with some transfers, are consolidated. Accounts *RdC* (Rest-of-Canada) and *RdM* (Rest-of-the-World, outside Canada) bring together exports and imports of all regions. Finally, supraregional account *RdQ* (Rest-of-Quebec) gathers imports of all regions from other Quebec regions, and their exports towards other regions in Quebec; interregional flows are contained in separate tables, one per product.

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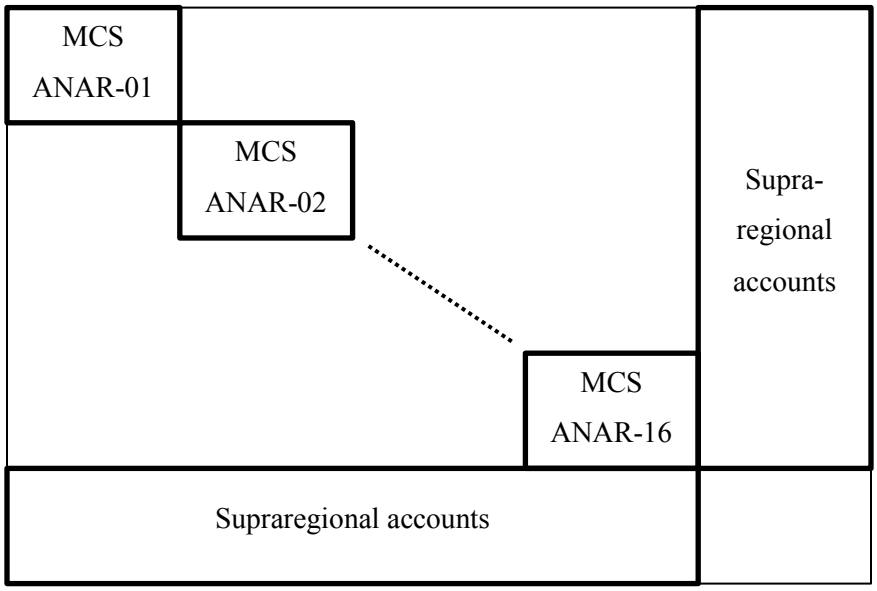
6 Sources used are CANSIM Tables 031-0004, 381-0023 and 381-0031, as well as the following reports: Applied Research Associates (2008), Deloitte&Touche (2012) and Ministère des transports du Québec (2012).

Figure 1 – Layout of the multiregional SAM



2.2.1 GDP by region and by industry

To assemble regional SAMs, and then a multiregional SAM, we first used data on regional GDP by industry published by ISQ.[7] These data are published by administrative region (AR), and by census metropolitan area (six CMAs and non-metropolitan Quebec). They were combined by adding and subtracting to obtain GDP by industry for each analytical region (ANAR; see Appendix 3).

Given their level of detail, these tables inevitably have confidential cells. Missing values have been estimated separately for ARs and CMAs from sums of missing values in each industry for all regions, and in each region for all industries, applying a relevant *a priori* distribution and the RAS adjustment technique. Estimation proceeded by blocks of industries, then the results were combined for each ANAR. For manufacturing, some indicators have been drawn from Statistics Canada's Annual Survey of Manufacturing and Logging Industries.

2.2.2 SAM

Next, GDP by industry and by ANAR was used as an allocator for IO data (intermediate demand for products and output). Household consumption was distributed among ANARs in proportion to ISQ's regional disposable income, which is tantamount to assume that the average propensity to consume is uniform, and that the structure of consumption expenditures is everywhere the same. Final demand by

7 On ISQ's regional GDP estimation method, see Lemelin and Mainguy (2009a, 2009b et 2008).

NPISHs was allocated to regions according to the amount of transfers from households to NPISHs. Final demand by public administrations (industries 40-44) is given by the value of their industry outputs for all levels of government taken together. Regarding investment expenditures, we used ISQ-published data on capital and repair expenditures from Statistics Canada's Annual Capital and Repair Expenditures Survey, supplemented with building-permits data. Intermediate and final demand estimated in that way was then converted to market prices assuming that margin rates and the rates of taxes on products are uniform across Quebec.[8]

Wages and mixed income received in each ANAR are calculated using ISQ data on disposable income by region. The discrepancy between wages received and paid according to value added by industry was attributed to commuting and shifted to a supraregional account. Corporate capital income and capital consumption expenditures (depreciation) are transferred to supraregional accounts *SOC* (Corporations) and *INV* (accumulation) respectively. The incomes of local administations (including school boards) were distributed among regions using allocators constructed from data of the *Ministère des affaires municipales et de l'occupation du territoire* (MAMOT), except for transfers received from the Provincial government (essentially, school board funding), which have been allocated according to public expenditures on education. Household income taxes are drawn from ISQ regional personal income data, assuming constant federal and provincial tax shares. Household transfers to NPISHs and to the RoW, as well as transfers received by households (except property income received from corporations) are also taken directly from ISQ regional personal income data. The rest of transfers has been established by applying proportionality rules, subject to accounting identities.

The previously established final demand of public administrations was disaggregated by level of government, either according to proportions in the whole-Quebec SAM, or using employment and salary data from Statistics Canada (CANSIM), the Federal and Quebec Treasury Boards, and, for local government, MAMOT.

2.3 ELABORATION OF INTERREGIONAL TRADE FLOWS

After building the regional SAMs, regional domestic production and demand for each product is known. The difference is the region's net exports. But regional data contains no information on crosshauling, regarding neither the origin of regional imports, or the destination of regional exports. Regional science literature is replete with writings about that difficulty, and there exists a large number of non-survey methods to construct exchange flows on the basis of different models.

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8 We reckon that the assumption of uniform transport margin rates is not realistic. We have accepted it for the time being, for want of indicators that would allow to modulate them according to geography.

For our part, we created interregional trade flows using a gravity model. Formally, the problem is, for each product, to fill in an origin-destination matrix whose marginal totals are known from the multiregional SAM. Origin (row) totals are given by regional outputs and by the volume of Quebec's imports from the RoC and the RoW; destination (column) totals are given by regional domestic demand and Quebec's exports to the RoC and the RoW.

2.3.1 Gravity model

The gravity model, in its so-called structural form (Head and Mayer, 2015) is summarized in the following equation

$$F_{i,j} = \frac{X_i}{\Omega_i}\frac{Q_j}{\Phi_j}\tau_{i,j} \qquad \text{[iii001]}$$

where

$F_{i,j}$: exports from region of origin i to region of destination j

$Q_j = \sum_i F_{i,j}$: domestic demand in destination region j

$X_i = \sum_j F_{i,j}$: output (supply) in region of origin i

$\tau_{i,j}$: Power of attraction between origin region i and destination region j

$\Phi_j = \sum_\ell \frac{\tau_{\ell,j} X_\ell}{\Omega_\ell}$: inward multilateral resistance factor of destination region j

$\Omega_i = \sum_\ell \frac{\tau_{i,\ell} Q_\ell}{\Phi_\ell}$: outward multilateral resistance factor of origin region i

Variables Φ_j et Ω_i are called "inward multilateral resistance" and "outward multilateral resistance" respectively. Terminologically, it is possible to reconcile the notion of resistance with those of accessibility and market potential. Indeed, if consumers in j have easy access to a large number of suppliers i, competition between the latter will confront each of them to greater resistance than if consumer access to suppliers is less easy. Reciprocally, if producers in i may offer their products on a large number of markets j, then the attractiveness of each market individually will be less. To summarize, the easier access residents have to suppliers, the greater the resistance that confronts each of the latter; and the greater the market potential of producers in a region, the better they can resist the power of buyers.

It should be noted that the $F_{i,j}$ in equation [iii001] represent the solution of a RAS adjustment of the *a priori* matrix given by the $\tau_{i,j}$, with X_i and Q_j for target marginal totals; the X_i/Ω_i are the row

multipliers, and the Q_j / Φ_j, the column multipliers. Since the RAS technique is equivalent to cross-entropy minimization, it follows logically that the distribution corresponding to the $F_{i,j}$ minimizes cross-entropy relative to the distribution corresponding to the $\tau_{i,j}$. Formally, let

$$f_{i,j} = \frac{F_{i,j}}{\sum_{h,k} F_{h,k}} \qquad \text{[iii002]}$$

$$\theta_{i,j} = \frac{\tau_{i,j}}{\sum_{h,k} \tau_{h,k}} \qquad \text{[iii003]}$$

Then [iii001] is the solution to the problem

$$\min_{f_{i,j}} \sum_{i,j} f_{i,j} \ln\left(f_{i,j} / \theta_{i,j}\right) \qquad \text{[iii004]}$$

2.3.2 Application to constructing interregional trade flows

The construction of interregional trade flows comprises several steps.

We have built interregional trade flows by applying a gravity model to *Municipalités régionales de comté* (MRC)[9] population data and road network distances between MRCs' principal towns. These are completed with data on several cities in Canada and the United States[10] regarding their weight in the trade network (population, corrected according to the volume of trade with Quebec), and their distances from each MRC.

First, we calculated flows between nodes of the trade network (principal towns of each MRC and external origin/destinations) using equation [iii001], with the following values:

$Q_i = X_i$: weight of node *i* (population, corrected according to trade volumes)

$$\tau_{i,j} = \frac{1}{d_{i,j}^{\sigma-1}} = d_{i,j}^{1-\sigma} \qquad \text{[iii005]}$$

$d_{i,j}$: distance between the principal town of MRC *i* et of MRC *j*

σ: elasticity of substitution between commodities of different origins

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9 In Quebec, there are 103 *Municipalités régionales de comté* (MRC; literally "Regional County Municipalities"). They are responsible for territorial planning.

10 We use the U.S. as an origin/destination which is "representative" of Quebec's trade with the RoW.

It can be seen from equation [iii005] that the larger elasticity σ, the greater the impediment of distance. With a high value of σ, ratios $\tau_{i,i}/\tau_{i,j}$ and $\tau_{i,i}/\tau_{j,i}$ for $j \neq i$ are large and the attractiveness of locally produced goods for buyers or of local outlets for sellers is much stronger than that of other origins or destinations. Consequently, cross-flows tend to be weak. In other words, if the local product is easily substituted to imports, then there is no reason to incur the cost of distance. That is why, to avoid overstating the importance of crosshauling, we set the value of σ at 2,5.[11]

The weights of the network nodes have been set according to population. For the MRCs, their weight is equal to their population, multiplied by the ratio of the sum for all commodities of the value of

[Quebec domestic demand + domestic output]

over the sum for all commodities of the value of

[Quebec domestic demand + domestic output + imports + exports]

As for RoC and RoW nodes, their population was multiplied by the ratio of the sum for all commodities of the value of

[exports + imports]

with the RoC or the RoW, depending on the case, over the sum for all commodities of the value of

[Quebec domestic demand + domestic output + imports + exports]

Next, we aggregated the nodes' weights and the trade flows constructed with the gravity model among MRCs and with RoC and RoW destinations, according to the geographical breakdown of the model: the 16 analytical regions, the RoC and the RoW. Inward and outward resistance variables Φ_j and Ω_i were aggregated as weighted sums. Finally, the $\tau_{i,j}$ indicators of attractiveness between aggregated regions were obtained by reversing equation [iii001], transposed to flows between the aggregate regions

$$\tau^R_{h,k} = \frac{\Omega^R_h}{X^R_h}\frac{\Phi^R_k}{Q^R_k}F^R_{h,k} \qquad \text{[iii006]}$$

where superscript *R* designates components of the aggregate model. Once the attractiveness factors have been calculated, one can reverse equation [iii005] to obtain synthetic distances between analytical regions:

$$d^R_{h,k} = \frac{1}{\left(\tau^R_{h,k}\right)^{1/(\sigma-1)}} = \left(\tau^R_{h,k}\right)^{-1/(\sigma-1)} \qquad \text{[iii007]}$$

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11 A higher value would have produced improbable results, because of a distorsion in the trade network caused by the proximity of Gatineau to Ottawa.

The interregional flow generation model consists mainly of attractiveness factors $\tau_{h,k}^{R}$. Given those attractiveness factors, for each product, we solve the simultaneous equation system given by

$$F_{h,k}^{R} = \frac{X_h^R}{\Omega_h^R}\frac{Q_k^R}{\Phi_k^R}\tau_{h,k}^{R} \qquad \text{[iii008]}$$

$$\Phi_k^R = \sum_h \frac{\tau_{h,k}^{R} X_h^R}{\Omega_h^R} \qquad \text{[iii009]}$$

$$\Omega_h^R = \sum_k \frac{\tau_{h,k}^{R} Q_k^R}{\Phi_k^R} \qquad \text{[iii010]}$$

where

Q_k^R : domestic demand in aggregate destination region *k* (exports to that region in the cases of RoC and RoW)

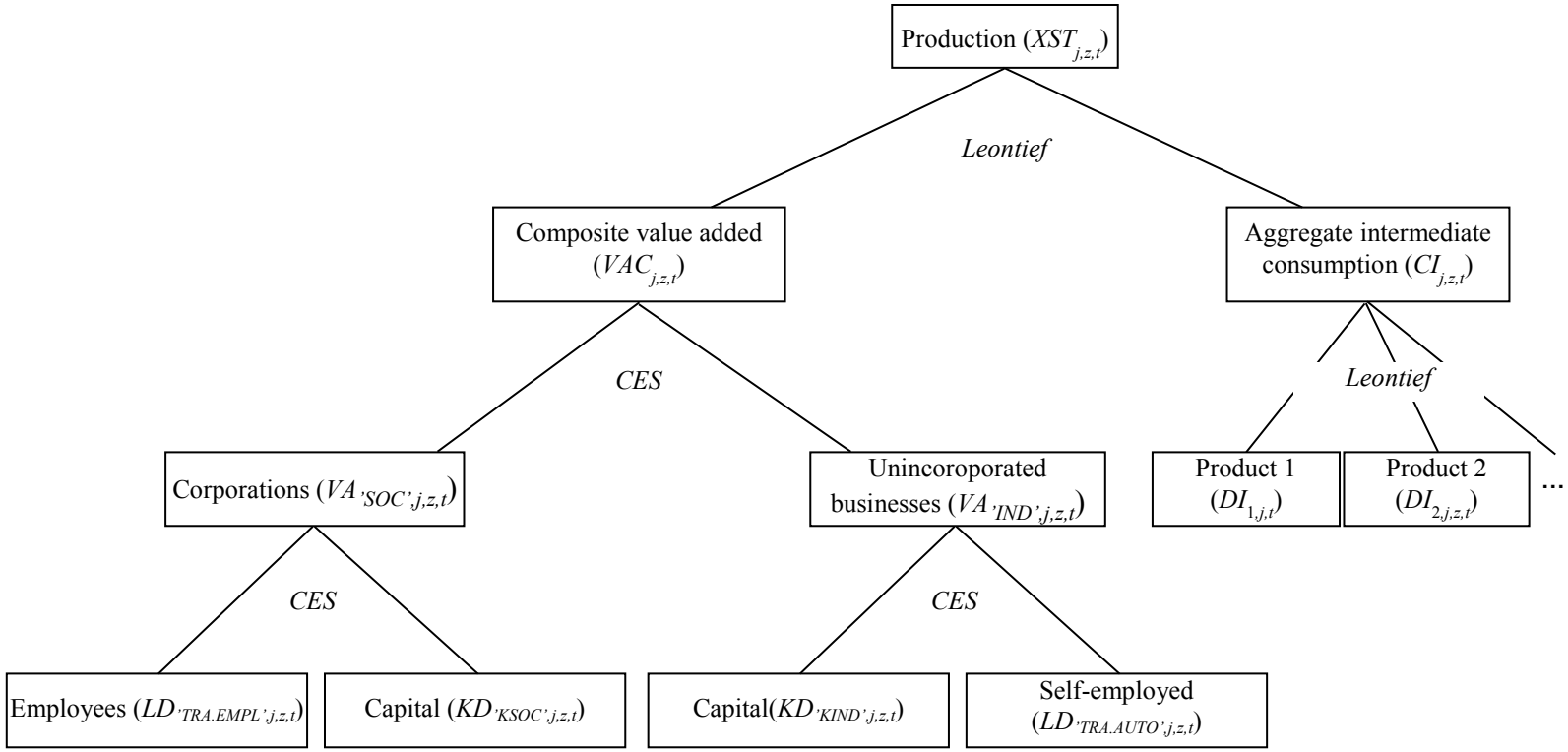
X_h^R : output (supply) in the aggregate origin region *h* (imports from the region inthe cases of RoC and RoW)

3. CGE model

The MEGBEC model was developed from the PEP-1-t model (Decaluwé *et al*., 2013). But PEP-1-t is a generic model, and we have altered it to take advantage of available data, as reflected in the SAM structure described above. A detailed description of the model (equations, sets, variables and parameters) is given in Appendix 1.

One of the features of our model is the distinction between corporations and unincorporated business, which is taken into account in the production structure (see Figure 2).

Figure 2 : Production structure



For want of information regarding the intermediate demand of corporations and unincorporated businesses, we assumed that the structure of intermediate consumption and the ratio of intermediate consumption to value added was the same in both cases. At the top level, therefore, the value of production is split according to a Leontief function into intermediate consumption and composite value added, which is a CES aggregate of the value added of corporations and of unincorporated businesses. Value added of corporations is a CES combination of labor supplied by employees (TRA.EMPL) and corporate capital (KSOC); value added of unincorporated businesses similarly combines the labor of self-employed workers (TRA.AUTO) and the capital of unincorporated businesses (KIND).

Each industry's output is then split between its different products. Total regional supply of each product is then sold on the different markets: Quebec, RoC and RoW. The volume supplied to the Quebec market is finally distributed among the sixteen analytical regions. That process is represented in the model through a nested CET structure as described by equations 67 to 75 in Appendix 1.

The model is recursive dynamic, and the evolution of productive capacity is determined by the accumulation of production factors through time. The supply of labor in each analytical region is exogenous and follows regional demographic perspectives. The stock of capital, by industry and region, is determined by the stock of capital in the preceding period, minus depreciation, to which are added the preceding period's investments. New capital created by investment is allocated among industries according to the PEP-1-t formulation, or nearly so. In fact, in PEP-1-t, discount rate (*IR*) is an endogenous variable that appears only in the calculation of the user cost of capital (equation 118 in Appendix A of Decaluwé *et al*., 2013b), and it has no other role than to adjust investment demand to preserve the balance between total investment demand and the available financing (savings) for private investment (equation 113). Therefore, in PEP-1-t, there is no link between discount rate *IR* and the rate or return of capital. Here, we use an equation for the discount rate (equation 117) that ties that variable to the rate of remuneration of capital. The discount rate corresponds to the weighted average of capital rates of remuneration, net of depreciation. Naturally, in so doing, we had to add an extra variable, factor Φ_t in equation 116, which plays the part of a rationing factor to maintain the private investment–savings balance (equation 113).

Investments aiming to increase the productive capacity of the public sector are exogenous, as are investments in road infrastructures. The latter, however, have no role in our model for the time being, in that they contribute to increasing the productive capacity of no industry.[12]

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[12] In many models that take into account the contribution of infrastructure to productivity, total factor productivity in the industry production functions depends on the stock of infrastructure.

Finally, the decomposition of investment among different asset categories (building construction, engineering construction, machinery and equipment...) follows a Leontief distribution, proportional to that of the reference period (equation 119) and total investment by asset category is obtained by adding up demand from all industries (equation 120).

Although individual product supply and demand balances are established at the regional level, most macroeconomic balances are implemented at the provincial level, in accordance with the SAM structure (supraregional accounts). So total investment in the province is determined by total savings (Appendix 1, equation 107), and the sum of regional labor supplies is equal to total labor demand by all industries in all regions (Appendix 1, equation 105). Likewise, exchanges with the rest of Canada and the rest of the World are accounted for at the provincial level, as are the corporations' account, the NPISH account and the property income account. Also, macro aggregates (GDP, household consumption, etc.) and price indexes are computed both at the regional and provincial levels.

Part of the activities of different orders of government are established at the regional scale: current expenditures on goods and services, investment expenditures, transfers to and from households and revenue from indirect taxation. For higher orders of government (provincial, federal, RRQ and CPP), the balance of regional transactions is transferred to the corresponding supraregional account. At that level, the activities of public administrations consist of the following: transfers to and from businesses (corporations and NPISHs), property income paid and received, and high-order intergovernmental transfers. In the SAM, the resulting balance is in line with the value of each order of government's savings according to the income and expenditure accounts. Equations 22 to 42 in Appendix 1 describe these relations.

The closure rules we use are quite similar to those of PEP-1-t. Per capita public investment expenditures are fixed in real terms, as are current public expenditures. The total value of investments in each period is endogenous and determined by the sum of savings (our model is "savings-driven"). Prices on external markets are exogenous and fixed. Quebec has two trading partners, the RoC and the RoW, both of which are characterized by fixed prices of imports from, and exports to Quebec; trade volumes adjust according to the Quebec internal prices in relation to these exogenous prices. For the RoW, we have maintained the hypothesis of a fixed current account balance (CAB). Regarding the balance with the RoC, we observe that leaving it free produced results that were less than convincing. As a matter of fact, given that the counterpart of a current account deficit (surplus) is positive (negative) foreign savings, any variation in the current account balance with the RoC results in a change of equal magnitude in total investment.

Therefore, abiding by the TANSTAAFL principle[13], we applied the same current account balance rule to the RoC as to the RoW: the nominal CAB to GDP ratio is constant. But introducing that extra constraint left the model one free variable short. So we introduced an "exchange rate" between Quebec and the RoC. This variable however is to be interpreted as an overall price ratio between Quebec and the RoC: since RoC prices are fixed, the endogenous Quebec-RoC "exchange rate" represents the overall evolution of Quebec prices relative to prices in the RoC. The rate of exchange with the RoW plays the part of numéraire.[14]

Model parametrization consists in determining the values of parameters that are consistent with the SAM data. But the SAM does not contain enough information to set all parameters, so that some, called "free parameters", must be assigned values by other means, generally values are drawn from the literature or econometrically estimated. Here, we have borrowed elasticity parameters from various sources. The tables in Appendix 5 contain the values of the main free parameters.

4. Simulation scenarios and analysis of results

4.1 Specification of the basic scenario and simulations

We have not yet developed a proper reference scenario as we have, for example, for model PEP-w-t (Lemelin et Robichaud, 2014). Our simulations are conducted starting from a "neutral" business-as-usual (BAU) scenario, where the supply of labor and some other exogenous variables grow according to regional demographic forecasts (total population by ANAR), without shock. Left on its own, so to speak, the model can run to 2060 and later.[15]

The goal of the simulations is to study the impact of recent developments in the Canadian and world economies on the economy of Quebec and of its regions. Specifically, we consider the impact of the fall in the prices of basic commodities. Among these, we have selected oil, which Quebec imports as crude for its refineries, and metals and minerals, for their importance in Quebec's economy (mining, aluminum smelting, *Plan Nord*...).

The sub-index of metals and minerals in the Bank of Canada's Basic Commodity Price Index (BCPI) covers: potash, aluminum, gold, nickel, iron, copper, silver, zinc and lead. Corresponding products in the model are 08-MIN_METAL (Minerals and metal concentrates) and 27-METAL_PREM (Primary metal

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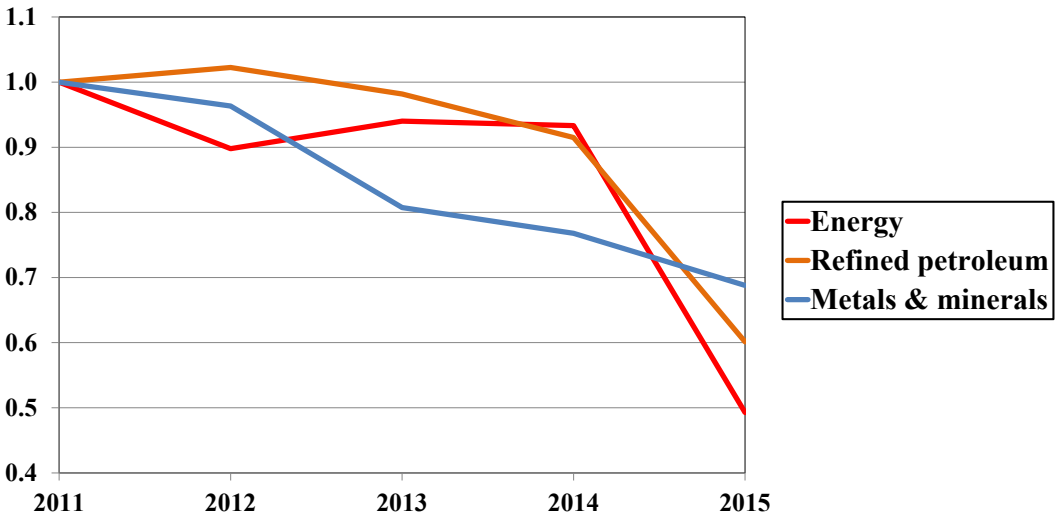
13 "There Ain't No Such Thing As A Free Lunch". Practically speaking, in simulations, if the CAB surplus of Quebec with the RoC diminishes, capital accumulation, and hence growth accelerate.

14 There are other options: see Lemelin (2015).

15 Results concerning such a distant future have no value in our opinion, but we consider the possibility of running the model for so long as a test of its robustness.

manufacturing). For oil, we have used two price indexes. For product 07-COMBUST (Mineral fuels), we took the Bank of Canada's BCPI sub-index for energy, consisting in: Crude Oil (WTI, Brent, Western Canada Crude), Natural Gas and Coal. As for product 23-PET_RAFF (Refined petroleum products, excluding chemicals), refining margins partly dissociate the prices of refined products from that of crude oil, so we have built an index from data published by the U.S. Energy Information Administration: we took the twelve-month average of gasoline wholesale and resale price by refiners over all of the United States, for 2011-2015 (last available year).[16] Figure 3 shows the evolution of world prices.

Figure 3 – Index of basic commodity prices (2011=1.0)



Sources : Bank of Canada (http://www.banqueducanada.ca/taux/indices-des-prix/ippb/)
U.S. Energy Information Administration (http://www.eia.gov/dnav/pet/pet_pri_refmg_dcu_nus_m.htm)

On that basis, we define three scenarios:

- SIM1 : fall in the world prices of energy (crude oil and refined petroleum products);
- SIM2 : fall in the world prices of metals and minerals;
- SIM3 : combination of the two preceding scenarios.

4.2 SIM1: FALL IN THE WORLD PRICES OF OIL

The first simulated shock is a fall in the world prices of crude oil and refined products, both imported and exported. In the model, corresponding products are 07-COMBUST (Mineral fuels) and 23-PET_RAFF (Refined petroleum products). Table 1 describes the evolution of world prices for these products in the simulation.

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16 U.S. Total Gasoline Wholesale/Resale Price by Refiners (Dollars per Gallon), Gasoline All grades : http://www.eia.gov/dnav/pet/pet_pri_refmg_dcu_nus_m.htm (site consulté 2016-04-12)

Table 1 : Variation of world prices of oil relative to preceding year

	07-COMBUST	23-PET_RAFF
2012	–10.2%	+2.2%
2013	–6.0%	–1.8%
2014	–6.7%	–8.5%
2015 to 2025	–50.7%	–39.9%

In order to better understand the impacts, let us first describe the place occupied by these products in the Quebec economy. Demand for product 07-COMBUST is almost entirely fulfilled by imports from the RoW, which represent a little more than 10% of the total value of Quebec's imports. Even though over 20% of local production is directed to exports (mainly to the RoC), those exports are a negligible share of the total value of Quebec's exports. So it is expected that a drop in world oil prices essentially benefits importers, with little impact on exporters.

The story is different with refined petroleum products. In fact, Quebec is a net exporter of refined products to the RoC, but a net importer from the RoW. All in all, the value of exports is roughly equal to the value of imports. Nearly half of the output of refined petroleum products is for the export market, and nearly half of demand in Quebec is satisfied from imports. Therefore, a shock to world prices will impact both supply and demand.

On the domestic market, purchases of mineral fuels are mostly for intermediate consumption. The refining industry (15-RAFFIN) by itself accounts for almost 80% of the total demand for product 07-COMBUST. As for refined petroleum, it is also largely for intermediate consumption, in almost every industry, but prominently in transport (close to 18% of total demand) and construction (close to 11%). Household final demand represents almost one third of total demand.

Table 2 : Supply and demand of oil products in Quebec, 2011

	07-COMBUST	23-PET_RAFF
Demand:		
Intermediate demand (M$)	13 246	10 377
Final demand (M$)	106	4 975
Share of intermediate demand	99.2%	67.6%
Supply:		
Share of product in the sales of industry 15-RAFFIN	2.4%	96.0%

Both goods are produced by the same industry, 15-RAFFIN. So a drop in world oil prices will have a two-way impact on the refining industry: it will diminish its production costs and, *ceteris paribus*, the value of its sales.

Regionally, the refining industry is principally located in the Montreal and Quebec CMAs. But in spite of it all, the relative importance of that industry in value added (GDP) remains small, both regionally and in Quebec as a whole.

4.2.1 Impact on the Quebec economy as a whole

Given that the products involved in the shock account for a greater share of total imports than exports, it is no wonder that the lower world prices of oil improve Quebec terms of trade with the RoW, and even with the RoC, as soon as 2012: the Fisher price index of exports rises relative to that of imports. Consequently, since the balance of trade is fixed in the model closure rules, the volume of exports to the RoW increases less than that of imports as time goes by; and although the opposite happens to the volume of trade with the RoC, the net effect is to free resources.

On the supply side, the fall in oil prices on the export market discourages foreign sales. Produces wanting to redirect their output to the domestic market will need to lower their local sales price. Symmetrically, the lower import price of product 07-COMBUST allows refiners to cut their production costs considerably. The lower price boosts demand, letting producers increase their output somewhat. Production initially directed to international exports is diverted to the domestic market and the RoC.

Table 3 : Impact on the supply of oil products
(% variation relative to the BAU scenario, 2025)

	07-COMBUST	23-PET_RAFF
Exports to the RoC		
Price	-56.1	-36.4
Volume	-38.2	1.2
Exports to the RoW		
Price	-59.2	-41.4
Volume	-42.2	-5.9
Sales in Quebec		
Price	-40.5	-32.0
Volume	-18.4	6.5
Total suply		
Price	-43.4	-34.8
Volume	-22.4	2.9

Table 4 : Impact on the demand for oil products
(% variation relative to the BAU scenario, 2025)

	07-COMBUST	23-PET_RAFF
Imports from the RoC		
Price	-46.9	-35.2
Volume	-3.8	10.7
Imports from the RoW		
Price	-50.7	-39.9
Volume	7.6	27.4
Purchases in Quebec		
Price	-40.5	-32.0
Volume	-18.4	6.5
Total demand		
Price	-50.0	-34.9
Volume	5.4	12.8

Given that two-thirds of the demand for refined products are used as inputs in other industries, the lower price of oil reduces the cost of inputs in several industries, increasing the share of factor incomes. Lower oil prices also release part of the households' consumption budget, increasing real consumption. As soon as the first year of the shock (2012), factor incomes rise (+2.4% for capital, +1.6% for labor), household demand increases (+1.6% in nominal terms, +0.4% real terms), and savings by all agents, and consequently investment, go up (+2.8% nominal, +1.4 real).

For 2011 and 2012, real GDP is the same in SIM1 as in the BAU because real GDP is nothing but a measure of the volume of production factors, capital and labor, which is the same in both scenarios for 2011 and 2012. But beginning in 2013, there appears a gap which can only result from a quicker capital accumulation in the SIM1 scenario (the supply of labor is identical in the two scenarios). And indeed, as mentioned in the preceding paragraph, the real value of savings is greater in SIM1 from 2012, so that beginning in 2013, when the new capital created in 2012 comes on line, the capital stock is larger than in the BAU. But how is it that the cost of investment increases less than savings? In fact, a significant share of equipment goods needed for investment is imported; in the simulation, the prices of these goods on the world market is unchanged, while domestic prices rise, so that the investment price index increases less than savings.

Table 5 : Impact on macroeconomic indicators
(% variation relative to the BAU scenario)

	GDP at basic prices			Household consumption			Gross fixed capital formation		
	Nom.	Real	Deflat.	Nom.	Real	Deflat.	Nom.	Real	Deflat.
2011	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2012	1.9	0.0	1.9	1.6	0.4	1.2	2.8	1.4	1.4
2013	0.6	0.2	0.4	0.6	0.5	0.1	0.5	0.4	0.1
2014	1.4	0.1	1.3	1.3	0.7	0.7	1.6	0.7	0.8
2015	11.6	0.1	11.4	10.3	3.7	6.4	14.6	6.2	7.9
2016	10.9	0.7	10.2	10.0	4.3	5.4	13.3	6.5	6.4
2017	10.9	0.9	9.9	10.0	4.4	5.3	13.5	6.9	6.2
2018	10.8	1.1	9.6	9.9	4.6	5.0	13.4	6.9	6.0
2019	10.7	1.3	9.3	9.8	4.7	4.8	13.3	7.0	5.9
2020	10.7	1.4	9.1	9.7	4.9	4.6	13.2	7.0	5.8
2021	10.6	1.6	8.9	9.6	5.0	4.4	13.1	7.0	5.7
2022	10.5	1.7	8.7	9.6	5.1	4.3	13.0	7.1	5.6
2023	10.5	1.8	8.6	9.5	5.2	4.1	13.0	7.1	5.5
2024	10.4	1.9	8.4	9.5	5.3	4.0	12.9	7.1	5.5
2025	10.4	1.9	8.3	9.4	5.4	3.9	12.8	7.1	5.4

In the course of time, growth in the stock of capital makes labor relatively scarcer, and its price rises. Moreover, increased economic activity generates more revenue for governments, and smaller deficits. Savings by corporations and households also contribute to higher total savings. Eventually, the gap between the two scenarios is widened by a feedback effect of growth on investment through RoW savings. Indeed, given that the trade balances with the RoW and the RoC are fixed proportions of GDP at basic prices, and given that their sum is negative (positive external savings), any rise in nominal GDP brings about an increase in foreign savings.

To summarize, our model predicts that the impact of the drop in world prices of crude oil and refined petroleum products will have a positive effect on the economy, even if only microeconomic impacts on resource allocation are taken into account. In a future version of the model, with unemployment, there could be an additional, macroeconomic impact, through a reduction in the rate of unemployment and a move towards full employment of labor.

4.2.2 Regional impacts

The refining industry (15-RAFFIN) is highly concentrated geographically. It generates about 55% of its value added in the Montréal AR (ANAR-01), and close to 27% in the Quebec CMA (ANAR-05). The transport industry (29-TRANSPORT), a prime user of refined petroleum, is relatively concentrated in Montreal (46% of its value added, compared to 34% for all industries taken together).

Table 6 : Regional impacts
(% variation relative to the BAU scenario, 2025)

	Real GDP	Real GFCF	Labor demand	Contribu-tion to Quebec GDP
01-RA Montréal	1.9	6.9	0.0	0.00
02-RA Laval	2.2	8.2	0.5	0.02
03-Reste RMR Montréal	2.2	7.5	0.1	0.02
04-Péri-Montréal	2.1	8.2	0.1	0.00
05-RMR Québec	1.8	6.6	0.1	0.03
06-Péri-Québec	1.8	6.5	-0.5	-0.02
07-RMR Gatineau	1.5	6.6	0.4	0.03
08-Péri-Gatineau	2.4	8.6	0.1	0.00
09-RMR Sherbrooke	1.6	6.9	-0.2	0.00
10-Péri-Sherbrooke	1.7	7.6	-0.8	-0.01
11-RMR Trois-Rivières	2.0	7.2	0.1	0.00
12-Péri-Trois-Rivières	1.9	7.0	-0.2	-0.01
13-RMR Saguenay	1.6	7.0	-0.4	-0.01
14-Péri-Saguenay	1.7	6.8	-0.4	-0.01
15-Reste du Nord	1.8	5.5	-0.7	-0.06
16-Est	1.8	5.7	-0.1	-0.01
Quebec	**1.9**	**7.1**	**-**	**-**

Having said that, we have seen that the impact of the drop in world energy prices was rather diffuse in the economy between industries, and the same is true geographically: the spatial distribution of economic activity is little changed from the oil price shock. All regions benefit from a rise in GDP beginning in 2013. And, from one region to another, the same industries, roughly speaking, contribute positively or negatively to the SIM1-BAU difference in real GDP.

4.3 SIM2: SHOCK ON THE WORLD PRICES OF METALS AND MINERALS

The second simulated shock consists in a drop in the world prices of metals and minerals, both imported and exported. In the Bank of Canada's BCPI, metals and minerals include: potash, aluminum, gold, nickel, iron, copper, silver, zinc and lead. In the model, these are 08-MIN_METAL (Minerals and metal concentrates) and 27-METAL_PREM (Primary metal manufacturing). Table 7 describes the price shock.

Table 7 : Variation in the world prices of metals and minerals relative to preceding year

2012	-3.7%
2013	-19.3%
2014	-23.2%
2015 to 2025	-31.2%

To better understand the impacts of a drop in the world prices of metals and minerals, let us take a look at the place they occupy in the Quebec economy. Seventy-nine percent of the production of industry 05-MINES consists of product 08-MIN_METAL, of which it is the sole producer. That product is 85% exported (39% to the RoC, and 46% to the RoW); on the domestic market, industry 19-METAL_PREM (Primary metal manufacturing) accounts for almost 90% of demand. Product 27-METAL_PREM represents 92% of the output of its principal supplier, 19-METAL_PREM, and it is 77% exported (66% to the RoW, and 11% to the RoC); the domestic market absorbs the rest, half of it as an input for the same 19-METAL_PREM industry. So mining and primary metal manufacturing are by far the industries hardest hit by the drop in the world price of their product. In addition, exports of these two products alone account for nearly 20% of Quebec's total exports abroad, and nearly 8% of its exports to the RoC. Consequently, a shock on world export prices will also have a strong impact on external trade.[17]

Table 8 : Supply and demand of metals and minerals in Quebec, 2011

	08-MIN_METAL	27-METAL_PREM
Demand:		
Intermediate demand (M$)	3 140	13 430
Final demand (M$)	129	762
Share of intermediate demand	96.0%	94.6%
Supply:		
Share of the product in the sales of industry 05-MINES	78.9%	
Share of the product in the sales of industry 19-METAL_PREM		92.4%

4.3.1 Impact on the Quebec economy as a whole

The drop in the world prices of metals deteriorates Quebec's terms of trade with the RoW and even with the RoC as soon as 2012: the Fisher price index of exports falls relative to that of imports. Consequently, the volume of exports increases faster than that of imports as time goes by; the same occurs with the volume of trade with the RoC. It follows that maintaining the current account balances drains more resources.

As expected, exports of metals and minerals decrease markedly, entailing a reduction in production too. Besides, since product 08-MIN_METAL enters in the manufacturing of 27-METAL_PREM, the decline of supply on the domestic market is even greater for minerals. And the world price drop has an impact on the domestic price, which is also more pronounced for minerals than for primary metals.

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17 On the import side, almost two-thirds of metals and minerals purchases are imported. But the relative importance of those imports in Quebec's total imports, around 6%, is much lower than for exports.

Table 9 : Impact on the supply of metals and minerals (% variation relative to the BAU scenario, 2025)

	08-MIN_METAL	27-METAL_PREM
Exports to the RoC		
Price	-7.9	-18.8
Volume	-24.1	-29.4
Exports to the RoW		
Price	-5.6	-16.0
Volume	-22.5	-28.5
Sales in Quebec		
Price	-29.9	-14.5
Volume	-45.8	-29.2
Total supply		
Prix	-10.4	-16.2
Volume	-26.7	-28.8

The decline in primary metals output results in a fall in the demand for minerals. As shown in Table 10, not only is demand weaker, but producers reduce their domestic purchases in favor of imports which have become relatively less costly. Regarding product 27-METAL_PREM, although the price of competing imports has fallen, the domestic cost of production has benefitted from the drop of minerals prices. So while we observe a reduction in the volume of imports demanded, it is however less than that of the demand for domestic products.

Table 10 : Impact the demand for metals and minerals (% variation relative to the BAU scenario, 2025)

	08-MIN_METAL	27-METAL_PREM
Imports from the RoC		
Price	-34.4	-34.4
Volume	-1.0	1.1
Imports from the RoW		
Prix	-31.2	-31.2
Volume	-39.5	-4.2
Purchases in Quebec		
Price	-29.9	-14.5
Volume	-45.8	-29.2
Total demand		
Price	-32.3	-26.5
Volume	-27.2	-13.1

Table 11 : Impact on macroeconomic indicators
(% variation relative to the BAU scenario)

	GDP at basic prices			Household consumption			Gross fixed capital formation		
	Nom.	Real	Deflat.	Nom.	Real	Deflat.	Nom.	Real	Deflat.
2011	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2012	-0.8	0.0	-0.8	-0.7	-0.1	-0.6	-1.1	-0.4	-0.7
2013	-4.1	-0.1	-4.0	-3.6	-0.7	-2.8	-5.5	-2.2	-3.4
2014	-4.7	-0.3	-4.4	-4.1	-1.0	-3.2	-6.3	-2.8	-3.7
2015	-6.2	-0.4	-5.8	-5.5	-1.3	-4.2	-8.4	-3.8	-4.8
2016	-6.0	-0.7	-5.4	-5.3	-1.3	-4.0	-8.2	-3.9	-4.4
2017	-5.9	-0.8	-5.1	-5.2	-1.4	-3.9	-8.1	-4.0	-4.3
2018	-5.8	-0.9	-4.9	-5.2	-1.4	-3.8	-8.0	-4.0	-4.2
2019	-5.8	-1.0	-4.8	-5.1	-1.5	-3.7	-7.9	-4.0	-4.1
2020	-5.7	-1.1	-4.7	-5.1	-1.5	-3.6	-7.9	-4.0	-4.0
2021	-5.7	-1.2	-4.5	-5.1	-1.6	-3.5	-7.8	-4.0	-3.9
2022	-5.6	-1.3	-4.4	-5.0	-1.6	-3.5	-7.8	-4.0	-3.9
2023	-5.6	-1.3	-4.3	-5.0	-1.7	-3.4	-7.7	-4.0	-3.8
2024	-5.6	-1.4	-4.3	-5.0	-1.7	-3.3	-7.7	-4.0	-3.8
2025	-5.5	-1.4	-4.2	-4.9	-1.7	-3.2	-7.6	-4.0	-3.8

Throughout the 2012-2025 period, a widening gap appears between real GDPs in the SIM2 and BAU scenarios. To summarize, at the 2025 horizon, industries that contribute most to the real GDP gap are the following:

19-METAL_PREM	–34.0%
05-MINES	–28.4%
07-CONSTRU	–18.4%

Together, these three industries represent over 80% of the real GDP loss.

The mechanism whereby real GDP in SIM2 slips away from its BAU level is the same as in scenario SIM1, but reversed. The real value of savings is less in SIM2 than in the BAU scenario, and that slows capital accumulation.

4.3.2 Regional impacts

In our geographical breakdown, the spatial concentration of mining (10-MINES) is very high: it generates 83% of its value added in the large Reste-du-Nord region (ANAR-15), where it is 28% of GDP. As for industry 19-METAL_PREM, it is not concentrated to the same extent as 10-MINES or 15-RAFFIN. But it counts for a substantial share of GDP in some of the regions where it concentrates. That is particularly true in the Saguenay CMA (ANAR-13: 14% of GDP) and in the neighboring region of Peri-Saguenay (ANAR-14: 11%).

Overall, by 2025, all regions suffer a loss of GDP relative to the BAU scenario, even if some of them gain in the beginning (Montreal up to 2017, the rest of the Montreal CMA up to 2016, and a handful of other

regions in the very early years of the scenario). And, just like in SIM1, it is the same industries which, from one region to another, contribute positively or negatively to the real GDP gap between SIM1 and the BAU.

In spite of the fact that the shock hits some regions (Saguenay, Reste-du-Nord) harder than others, the distribution of Quebec's GDP among regions is not very different between scenarios, as can be seen in Table 12. Such relative stability is probably partly due to the fact that the preliminary version of the model presented here is better fit for analyzing impacts on production, because income distribution is independent of where production occurs; it follows that no regional impact multiplier comes into play, whether the shock be positive or negative.

Table 12 : Regional impacts
(% variation relative to the BAU scenario, 2025)

	Real GDP	Real GFCF	Labor demand	Contribu-tion to Quebec GDP
01-RA Montréal	-0.59	-2.97	0.45	0.27
02-RA Laval	-0.81	-3.58	0.22	0.02
03-Reste RMR Montréal	-0.72	-3.23	0.53	0.11
04-Péri-Montréal	-1.44	-4.34	0.03	-0.01
05-RMR Québec	-0.75	-2.97	0.21	0.05
06-Péri-Québec	-1.06	-3.52	0.67	0.01
07-RMR Gatineau	-0.48	-2.75	0.08	0.01
08-Péri-Gatineau	-0.91	-3.91	0.39	0.00
09-RMR Sherbrooke	-0.47	-3.09	0.68	0.02
10-Péri-Sherbrooke	-0.81	-3.76	1.00	0.01
11-RMR Trois-Rivières	-2.56	-4.85	-0.91	-0.02
12-Péri-Trois-Rivières	-0.83	-3.69	0.60	0.03
13-RMR Saguenay	-6.65	-9.30	-4.10	-0.12
14-Péri-Saguenay	-4.83	-7.92	-2.71	-0.04
15-Reste du Nord	-9.11	-12.00	-4.92	-0.34
16-Est	-1.35	-3.28	0.04	0.00
Quebec	**-1.39**	**-4.04**	**-**	**-**

4.4 SIM3: COMBINED SCENARIO: FALL IN THE WORLD PRICES OF OIL AND OF METALS AND MINERALS

4.4.1 Impact on the Quebec economy as a whole

The question raised in this third scenario is: which of the two shocks has the stronger impact? Will the positive impact of the drop in oil prices overtake the negative impact of the fall in the prices of metals and minerals?

For Quebec as a whole, the negative effect initially dominates, and increasingly so, until 2015, after which the gap between SIM3 and the BAU scenario becomes positive and growing up to the 2025 horizon, when GDP is about 0.6% higher.

Table 13 : Impact on macroeconomic indicators
(% variation relative to the BAU scenario)

	GDP at basic prices			Household consumption			Gross fixed capital formation		
	Nom.	Real	Deflat.	Nom.	Real	Deflat.	Nom.	Real	Deflat.
2011	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2012	1.1	0.0	1.1	0.9	0.3	0.6	1.7	0.9	0.7
2013	-3.5	0.1	-3.6	-3.0	-0.3	-2.7	-5.1	-1.9	-3.2
2014	-3.3	-0.2	-3.1	-2.8	-0.3	-2.5	-4.8	-2.0	-2.8
2015	5.0	-0.3	5.3	4.6	2.5	2.1	5.7	2.6	3.0
2016	4.6	0.0	4.6	4.4	3.0	1.3	4.7	2.7	1.9
2017	4.7	0.1	4.6	4.4	3.1	1.3	5.0	3.0	1.9
2018	4.7	0.2	4.4	4.4	3.2	1.1	4.9	3.1	1.8
2019	4.6	0.3	4.3	4.3	3.3	1.0	4.9	3.1	1.7
2020	4.6	0.4	4.2	4.3	3.4	0.9	4.8	3.1	1.7
2021	4.6	0.4	4.1	4.2	3.4	0.8	4.8	3.1	1.6
2022	4.5	0.5	4.0	4.2	3.5	0.7	4.7	3.1	1.6
2023	4.5	0.5	3.9	4.2	3.5	0.6	4.7	3.1	1.6
2024	4.5	0.6	3.9	4.2	3.6	0.6	4.7	3.1	1.5
2025	4.5	0.6	3.8	4.2	3.6	0.5	4.7	3.1	1.5

4.4.2 Regional impacts

In the end, regions that loose are those that are directly hit by the shock on the prices of metals and minerals: the Saguenay CMA (–4.9% relative to BAU), and Reste-du-Nord (–7.4%), as well as, marginally, the Trois-Rivières CMA (–0.5%). Roughly speaking, given that industries evolve similarly from one region to another, the fate of regions depends on the industrial composition of their economy.

Table 14 : Regional impacts
(% variation relative to the BAU scenario, 2025)

	Real GDP	Real GFCF	Labor demand	Contribu-tion to Quebec GDP
01-RA Montréal	1.47	4.14	0.50	0.26
02-RA Laval	1.52	4.75	0.68	0.04
03-Reste RMR Montréal	1.55	4.37	0.58	0.13
04-Péri-Montréal	0.77	3.99	0.10	0.00
05-RMR Québec	1.14	3.74	0.31	0.08
06-Péri-Québec	0.78	3.08	0.06	0.00
07-RMR Gatineau	1.06	3.87	0.44	0.05
08-Péri-Gatineau	1.48	4.70	0.43	0.00
09-RMR Sherbrooke	1.16	3.83	0.42	0.02
10-Péri-Sherbrooke	0.95	3.85	0.10	0.00
11-RMR Trois-Rivières	-0.50	2.38	-0.79	-0.02
12-Péri-Trois-Rivières	1.11	3.41	0.32	0.01
13-RMR Saguenay	-4.93	-2.52	-4.33	-0.12
14-Péri-Saguenay	-3.06	-1.21	-3.05	-0.04
15-Reste du Nord	-7.36	-6.90	-5.52	-0.39
16-Est	0.51	2.48	-0.11	0.00
Quebec	**0.64**	**3.11**	**-**	**-**

5. Conclusion

We have elaborated a 2011 SAM for Quebec, broken down into sixteen analytical regions. We have also generated interregional trade flows by running simulations with a gravity model. On that basis, we have built a multiregional recursive dynamic CGE model, MEGBEC, which can simulate the evolution of the economy of Quebec and of its regions. We have used the MEGBEC model to simulate the impact of recent fluctuations of world prices of oil and of metals and minerals. The simulation results show that a drop in the price of oil has a positive impact on the Quebec economy and benefits all regions in similar proportions. To the contrary, a fall in the prices of metals and minerals has a negative impact on the economy of Quebec as a whole, and the shock is felt differently across regions, with the Saguenay and the North loosing most.

What has been presented here should be considered but a first, albeit major, step in the development of the MEGBEC model. Much has been accomplished, much remains to be done. Among the tasks ahead, let us mention: a tighter link between the remuneration of factors in a region (especially labor), and regional household income; a better specification of the reference scenario; taking account of unemployment on the labor market; modeling the impact of infrastructure investments on the economy; differenciating transport margins over space; expliciting RoC supplies and demands; applying consumption expenditure

structures specific to each region; taking into account the effect of differences in asset depreciation rates on the of the structure of capital; including welfare indicators... There is still plenty to do!

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APPENDIX 1 – DESCRIPTION OF THE MODEL: EQUATIONS, SETS, VARIABLES AND PARAMETERS

A1.1 Equations

A1.1.1 PRODUCTION

1. $VA_{j,z,t} = v_{j,j} \cdot XST_{j,z,t}$

2. $CI_{j,z,t} = io_{j,z} \cdot XST_{j,z,t}$

3. $$VAC_{j,z,t} = B^{VAC}_{j,z}\left[\beta^{VAC}_{j,z} \cdot VA_{'SOC',j,z,t}^{-\rho^{VAC}_{j}} + \left(1-\beta^{VAC}_{j,z}\right)\cdot VA_{'IND',j,z,t}^{-\rho^{VAC}_{j}}\right]^{-\frac{1}{\rho^{VAC}_{j}}}$$

4. $$VA_{'IND',j,z,t} = \left[\frac{\beta^{VAC}_{j,z}}{1-\beta^{VAC}_{j,z}} \cdot \frac{PVA_{'SOC',j,z,t}}{PVA_{'IND',j,z,t}}\right]^{\sigma^{VAC}_{j}} VA_{'SOC',j,z,t}$$

5. $$VA_{'SOC',j,z,t} = B^{VA}_{'SOC',j,z}\left[\beta^{VA}_{'SOC',j,z} \cdot LD_{'TRA.EMPL',j,z,t}^{-\rho^{VA}_{'SOC',j}} + \left(1-\beta^{VA}_{'SOC',j,z}\right)\cdot KD_{'KSOC',j,z,t}^{-\rho^{VA}_{'SOC',j}}\right]^{-\frac{1}{\rho^{VA}_{'SOC',j}}}$$

6. $$LD_{'TRA.EMPL',j,z,t} = \left[\frac{\beta^{VA}_{'SOC',j,z}}{1-\beta^{VA}_{'SOC',j,z}} \cdot \frac{R_{'KSOC',j,z,t}}{W_{'TRA.EMPL',t}}\right]^{\sigma^{VAC}_{j}} KD_{'KSOC',j,z,t}$$

7. $$VA_{'IND',j,z,t} = B^{VA}_{'IND',j,z}\left[\beta^{VA}_{'IND',j,z} \cdot LD_{'TRA.AUTO',j,z,t}^{-\rho^{VA}_{'IND',j}} + \left(1-\beta^{VA}_{'IND',j,z}\right)\cdot KD_{'KIND',j,z,t}^{-\rho^{VA}_{'IND',j}}\right]^{-\frac{1}{\rho^{VA}_{'IND',j}}}$$

8. $$LD_{'TRA.AUTO',j,z,t} = \left[\frac{\beta^{VA}_{'IND',j,z}}{1-\beta^{VA}_{'IND',j,z}} \cdot \frac{R_{'KIND',j,z,t}}{W_{'TRA.AUTO',t}}\right]^{\sigma^{VAC}_{j}} KD_{'KIND',j,z,t}$$

9. $DI_{i,j,z,t} = aij_{i,j,z} \cdot CI_{j,z,t}$

A1.1.2 INCOME AND SAVINGS

Households

10. $YH_{h,z,t} = YHL_{h,z,t} + YHK_{h,z,t} + YHTR_{h,z,t}$

11. $$YHL_{h,z,t} = \sum_{l} \lambda^{WL}_{h,l,z} \left(W_{l,t} \sum_{j,zj} LD_{l,j,zj,t} + W^{RDC}_{l,t} LD^{RDC}_{l,t} \right)$$

12. $$YHK_{h,z,t} = \sum_{k} \lambda^{RK}_{h,k,z} \left(\sum_{j,zj} R_{k,j,zj,t} KD_{k,j,zj,t} \right)$$

13. $$YHTR_{h,z,t} = \sum_{ag,zt} TR_{h,z,ag,zt,t} + RP^{RECU}_{h,z,t}$$

14. $$YDH_{h,z,t} = YH_{h,z,t} - \sum_{gvt} \left(TDH_{gvt,h,z,t} + TR_{gvt,z,h,z,t} \right)$$

15. $$CTH_{h,z,t} = YDH_{h,z,t} - SH_{h,z,t} - \sum_{agng,zt} TR_{agng,zt,h,z,t} - RP^{PAYE}_{h,z,t}$$

16. $$SH_{h,z,t} = PIXCON_{zs,t}{}^{\eta} sh0_{h,z,t} + sh1_{h,z,t} YDH_{h,z,t}$$

Firms

17. $$YF_{f,t} = YFK_{f,t} + YFTR_{f,t}$$

18. $$YFK_{f,t} = \sum_{k} \lambda^{RK}_{f,k,zs} \left(\sum_{j,z} R_{k,j,z,t} KD_{k,j,z,t} \right)$$

19. $$YFTR_{f,t} = \sum_{ag,zt} TR_{f,zs,ag,zt,t} + RP^{RECU}_{f,zs,t}$$

20. $$YDF_{f,t} = YF_{f,t} - \sum_{gvtsup} TDF_{gvtsup,f,t}$$

21. $$SF_{f,t} = YDF_{f,t} - CTF_{f,t} - \sum_{ag,zt} TR_{ag,zt,f,zs,t} - RP^{PAYE}_{f,zs,t}$$

Governments

22. $$YG_{gvt,z,t} = YGK_{gvt,z,t} + TDHT_{gvt,z,t} + TIPT_{gvt,z,t} + TPRCTS_{gvt,z,t} + YGTR_{gvt,z,t}$$

23. $$YGK_{gvt,z,t} = \sum_{k} \lambda^{RK}_{gvt,k,z} \left(\sum_{j,zj} R_{k,j,zj,t} KD_{k,j,zj,t} \right)$$

24. $$TDHT_{gvt,z,t} = \sum_{h} TDH_{gvt,z,h,t}$$

25. $$TIPT_{gvt,z,t} = \sum_{j} TIP_{gvt,j,z,t}$$

26. $$TPRCTS_{gvt,z,t} = TICT_{gvt,z,t} + TIMT_{gvt,z,t} + TIXT_{gvt,z,t}$$

27. $$TICT_{gvt,z,t} = \sum_{i} TIC_{gvt,i,z,t} + \sum_{j} TIS_{gvt,j,z,t}$$

28. $$TIMT_{gvt,z,t} = \sum_{i} TIM_{gvt,i,z,t}$$

29. $$TIXT_{gvt,z,t} = \sum_{i} TIX_{gvt,i,z,t}$$

30. $$YGTR_{gvt,z,t} = \sum_{ag,zt} TR_{gvt,z,ag,zt,t} + RP^{RECU}_{gvt,z,t}$$

31. $$TDH_{gvt,h,z,t} = PIXCON_{zs,t}^{\eta} \; ttdh0_{gvt,h,z,t} + ttdh1_{gvt,h,z,t} YH_{h,z,t}$$

32. $$TIP_{gvt,j,z,t} = ttip_{gvt,j,z,t} \cdot PP_{j,z,t} \cdot XST_{j,z,t}$$

33. $$TIC_{gvt,i,z,t} = ttic^{DF}_{gvt,i,z,t} \cdot \left[\left(PCOMP_{i,z,t} + \sum_{ij} PCOMP_{ij,z,t} \cdot tmrg^{DF}_{ij,i,z} \right) \cdot DF_{i,z,t} \right] + ttic^{DIT}_{gvt,i,z,t} \cdot \left[\left(PCOMP_{i,z,t} + \sum_{ij} PCOMP_{ij,z,t} \cdot tmrg^{DIT}_{ij,i,z} \right) \cdot DIT_{i,z,t} \right]$$

34. $$TIS_{gvt,j,z,t} = ttis_{gvt,j,z,t} \cdot PP_{j,z,t} \cdot XST_{j,z,t}$$

35. $$TIM_{gvt,i,z,t} = \sum_{row} ttim_{gvt,i,row,z,t} \cdot PWM_{i,row,z,t} \cdot e_{row,t} \cdot IM_{i,row,z,t}$$

36. $$TIX_{gvt,i,z,t} = \sum_{row} ttix_{gvt,i,row,z,t} \left(PE_{i,row,z,t} + \sum_{ij} PCOMP_{ij,z,t} \cdot tmrg^{X}_{ij,i,row,z} \right) EXD_{i,row,z,t}$$

37. $$SG_{gvt,z,t} = YG_{gvt,z,t} - \sum_{ag,zt} TR_{ag,zt,gvt,z,t} - G_{gvt,z,t} - RP^{PAYE}_{gvt,z,t}$$

38. $$YG_{gvtsup,zs,t} = TDFT_{gvtsup,t} + YGTR_{gvtsup,zs,t} + \sum_{z} SG_{gvtsup,z,t}$$

39. $$TDFT_{gvtsup,t} = \sum_{f} TDF_{gvtsup,f,t}$$

40. $$TDF_{gvtsup,f,t} = PIXCON_{zs,t}^{\eta} \cdot ttdf0_{gvtsup,f,t} + ttdf1_{gvtsup,f,t} \cdot YFK_{f,t}$$

41. $$YGTR_{gvtsup,zs,t} = \sum_{ag,zt} TR_{gvtsup,zs,ag,zt,t} + RP^{RECU}_{gvtsup,zs,t}$$

42. $$SG_{gvtsup,zs,t} = YG_{gvtsup,zs,t} - \sum_{ag,zt} TR_{ag,zt,gvtsup,zs,t} - RP^{PAYE}_{gvtsup,zs,t}$$

Exterior

43. $$YROW_{'RDM',t} = e_{'RDM',t} \sum_{i,z} PWM_{'i,RDM',t} \left(IM_{i,'RDM',z,t} + REX_{i,z,t} \right) + \sum_{ag,zt} TR_{'RDM',zs,ag,zt,t} + RP^{RECU}_{'RDM',zs,t}$$

44. $$YROW_{'RDC',t} = e_{'RDC',t} \sum_{i,z} PWM_{i,'RDC',t} \cdot IM_{i,'RDC',z,t} + \sum_{ag,zt} TR_{'RDC',zs,ag,zt,t} + RP^{RECU}_{'RDC',zs,t}$$

45. $$SROW_{'RDM',t} = YROW_{'RDM',t} - \sum_{i,z} (PE^{FOB}_{i,'RDM',t} \cdot EXD_{i,'RDM',z,t} + PE^{FOBR}_{i,t} \cdot REX_{i,z,t}) - \sum_{agd,zt} TR_{agd,zt,'RDM',,zst} - RP^{PAYE}_{'RDM',zs,t}$$

46. $$SROW_{'RDC',t} = YROW_{'RDC',t} - \sum_{i} PE^{FOB}_{i,'RDC',z,t} \cdot EXD_{i,'RDC',z,t} - \sum_{agd,zt} TR_{agd,zt,'RDC',zs,t} - RP^{PAYE}_{'RDC',zs,t} - \sum_{l} W^{RDC}_{l,t} LD^{RDC}_{l,t}$$

47. $$SROW_{row,t} = -CAB_{row,t}$$

48. $$\frac{CAB_{row,t}}{GDP^{BP}_{zs,t}} = \frac{CAB^{O}_{row}}{GDP^{BP.O}_{zs}}$$

Transfers

49. $$TR_{agng,zt,h,z,t} = \lambda^{TR}_{agng,zt,h,z} \cdot YDH_{h,z,t}$$

50. $$TR_{gvt,z,h,z,t} = PIXCON^{\eta}_{zs,t} \cdot tr0_{gvt,h,z,t} + tr1_{gvt,h,z,t} \cdot YH_{h,z,t}$$

51. $$TR_{ag,zt,f,zs,t} = \lambda^{TR}_{ag,zt,f,zs} \cdot YDF_{f,t}$$

52. $$TR_{ag,zt,gvt,ztj,t} = PIXCON_{zs,t}^{\ \eta} \cdot TR^{O}_{ag,zt,gvt,ztj} \cdot pop_{zt,t}$$

53. $$TR_{ag,zt,row,zs,t} = PIXCON_{zs,t}^{\ \eta} \cdot TR^{O}_{ag,zt,row,zs} \cdot pop_{zt,t}$$

Property income

54. $$RP^{PAYE}_{h,z,t} = \lambda^{RPP}_{h,z} \cdot CTH_{h,z,t}$$

55. $$RP^{PAYE}_{f,zs,t} = \lambda^{RPP}_{f,zs} \cdot YDF_{f,t}$$

56. $$RP^{PAYE}_{gvt,zt,t} = PIXCON_{zs,t}^{\ \eta} \cdot RP^{PAYE.O}_{gvt,zt} \cdot pop_{zt,t}$$

57. $$RP^{PAYE}_{row,zs,t} = PIXCON_{zs,t}^{\eta} \cdot RP^{PAYE.O}_{row,zs} \cdot pop_{zt,t}$$

58. $$RP^{RECU}_{ag,zt,t} = \lambda^{RPR}_{ag,zt} \sum_{agj,ztj} RP^{PAYE}_{agj,ztj,t}$$

A1.1.3 DEMAND

59. $$PDF_{i,z,t} \cdot C_{i,h,z,t} = PDF_{i,z,t} \cdot C^{MIN}_{i,h,z,t} + \gamma^{LES}_{i,h,z} \cdot \left(CTH_{h,z,t} - \sum_{ij} PDF_{ij,z,t} C^{MIN}_{ij,h,z,t} \right)$$

60. $$PDF_{i,z,t} \cdot INV_{i,a,z,t} = \gamma^{INV}_{i,a,z} \cdot IT^{A}_{a,z,t}$$

61. $$INVT_{i,z,t} = \sum_{a} INV_{i,a,z,t}$$

62. $$PDF_{i,z,t} \cdot CG_{i,gvt,z,t} = \gamma^{GVT}_{i,gvt,z} \cdot G_{gvt,z,t}$$

63. $$PDF_{i,z,t} \cdot CF_{i,f,z,t} = \gamma^{F}_{i,f,z} \cdot CTF_{f,z,t}$$

64. $$DF_{i,z,t} = \sum_{h} C_{i,h,z,t} + \sum_{gvt} CG_{i,gvt,z,t} + \sum_{f} CF_{i,f,z,t} + INVT_{i,z,t} + VSTK^{IN}_{i,z,t}$$

65. $$DIT_{i,z,t} = \sum_{j} DI_{i,j,z,t}$$

66. $$MRGN_{i,z,t} = \sum_{ij} tmrg^{DF}_{i,ij,z} \cdot DF_{ij,z,t} + \sum_{ij} tmrg^{DIT}_{i,ij,z} \cdot DIT_{ij,z,t} + \sum_{ij,row} tmrg^{X}_{i,ij,row,z} \cdot EXD_{ij,row,z,t} + \sum_{ij} tmrg^{XR}_{i,ij,z} \cdot REX_{ij,z,t}$$

A1.1.4 TRADE

67. $$XST_{j,z,t} = B^{XT}_{j,z} \left[\sum_{i} \beta^{XT}_{j,i,z} \cdot XS^{\rho^{XT}_{j}}_{j,i,z,t} \right]^{\frac{1}{\rho^{XT}_{j}}}$$

68. $$XS_{j,i,z,t} = \frac{XST_{j,z,t}}{\left(B^{XT}_{j,z}\right)^{1+\sigma^{XT}_{j}}} \left[\frac{P_{i,z,t}}{\beta^{XT}_{j} \cdot PT_{j,z,t}} \right]^{\sigma^{XT}_{j}}$$

69. $$XSI_{i,z,t} = \sum_{j} XS_{j,i,z,t}$$

70. $$XSI_{i,z,t} = B^{X}_{i,z}\left[\begin{array}{l}\beta^{X.RDM}_{i,z}\cdot EX^{\rho^{X}_{i}}_{i,'RDM',z,t}+\beta^{X.RDC}_{i,z}\cdot EX^{\rho^{X}_{i}}_{i,'RDC',z,t}\\+\left(1-\beta^{X.RDC}_{i,z}-\beta^{X.RDM}_{i,z}\right)\cdot DS^{\rho^{X}_{i}}_{i,z,t}\end{array}\right]^{\frac{1}{\rho^{X}_{i}}}$$

71. $$EX_{i,'RDM',z,t} = \left[\frac{1-\beta^{X.RDM}_{i,z}-\beta^{X.RDC}_{i,z}}{\beta^{X.RDM}_{i,z}}\cdot\frac{PE_{i,'RDM',z,t}}{PL_{i,z,t}}\right]^{\sigma^{X}_{i}} DS_{i,z,t}$$

72. $$EX_{i,'RDC',z,t} = \left[\frac{1-\beta^{X.RDM}_{i,z}-\beta^{X.RDC}_{i,z}}{\beta^{X.RDC}_{i,z}}\cdot\frac{PE_{i,'RDC',z,t}}{PL_{i,z,t}}\right]^{\sigma^{X}_{i}} DS_{i,z,t}$$

73. $$EX_{i,'RDC',z,t} = \left[\frac{\beta^{X.RDM}_{i,z}}{\beta^{X.RDC}_{i,z}}\cdot\frac{PE_{i,'RDC',z,t}}{PE_{i,'RDM',z,t}}\right]^{\sigma^{X}_{i}} EX_{i,'RDM',z,t} \quad (si\ DS=0)$$

74. $$DS_{i,z,t} = B^{XQ}_{i,z}\left[\sum_{zj}\beta^{XQ}_{i,z,zj}\cdot OPQ^{\rho^{XQ}_{i,z}}_{i,z,zj,t}\right]^{\frac{1}{\rho^{XQ}_{i,z}}}$$

75. $$OPQ_{i,z,zj,t} = \frac{DS_{i,z,t}}{\left(B^{XQ}_{i,z}\right)^{1+\sigma^{XQ}_{i,z}}}\left[\frac{PPQ_{i,z,zj,t}}{\beta^{XQ}_{i,z,zj}\cdot PL_{i,z,t}}\right]^{\sigma^{XQ}_{i,z}}$$

76. $$EXD_{i,row,z,t} = EXD^{O}_{i,row,z}\cdot pop_{t}\cdot\left(\frac{e_{row,t}\cdot PWX_{i,row,z,t}}{PE^{FOB}_{i,row,z,t}}\right)^{\sigma^{XD}_{i,row}}$$

77. $$REX_{i,z,t} = REX^{O}_{i,z}\cdot pop_{t}\cdot\left(\frac{e_{'RDM',t}\cdot PWXR_{i,z,t}}{PE^{FOBR}_{i,z,t}}\right)^{\sigma^{REX}_{i}}$$

78. $$Q_{i,z,t} = B^{M}_{i,z}\left[\begin{array}{l}\beta^{M.RDM}_{i,z}\cdot IM^{-\rho^{M}_{i}}_{'RDM',i,z,t}+\beta^{M.RDC}_{i,z}\cdot IM^{-\rho^{M}_{i}}_{'RDC',i,z,t}\\+\left(1-\beta^{M.RDM}_{i,z}-\beta^{M.RDC}_{i,z}\right)\cdot DD^{-\rho^{M}_{i}}_{i,z,t}\end{array}\right]^{\frac{-1}{\rho^{M}_{i}}}$$

79. $$IM_{'RDM',i,z,t} = \left[\frac{\beta^{M.RDM}_{i,z}}{1-\beta^{M.RDM}_{i,z}-\beta^{M.RDC}_{i,z}}\cdot\frac{PL_{i,z,t}}{PM_{'RDM',i,z,t}}\right]^{\sigma^{M}_{i}} DD_{i,z,t}$$

80. $$IM_{'RDC',i,z,t} = \left[\frac{\beta_{i,z}^{M.RDC}}{1 - \beta_{i,z}^{M.RDM} - \beta_{i,z}^{M.RDC}} \cdot \frac{PL_{i,z,t}}{PM_{'RDC',i,z,t}} \right]^{\sigma_i^M} DD_{i,z,t}$$

81. $$DD_{i,z,t} = B_{i,z}^{MQ} \left[\sum_{zj} \beta_{i,zj,z}^{MQ} \cdot DPQ_{i,zj,z,t}^{-\rho_{i,z}^{MQ}} \right]^{-\frac{1}{\rho_{i,z}^{MQ}}}$$

82. $$DPQ_{i,zj,z,t} = \left(B_{i,z}^{MQ}\right)^{\sigma_{i,z}^{MQ}-1} \cdot DD_{i,z,t} \cdot \left[\frac{\beta_{i,zj,z}^{MQ} \cdot PD_{i,z,t}}{PPQ_{i,zj,z,t}} \right]^{\sigma_{i,z}^{MQ}}$$

A1.1.5 PRICES

Production

83. $$PP_{j,z,t} = \frac{PVAC_{j,z,t} \cdot VAC_{j,z,t} + PCI_{j,z,t} \cdot CI_{j,z,t}}{XST_{j,z,t}}$$

84. $$PT_{j,z,t} = \left(1 + \sum_{gvt} ttip_{gvt,j,z,t} + \sum_{gvt} ttis_{gvt,j,z,t} \right) PP_{j,z,t}$$

85. $$PCI_{j,z,t} = \frac{\sum_{i} PDI_{i,z,t} \cdot DI_{i,j,z,t}}{CI_{j,z,t}}$$

86. $$PVAC_{j,z,t} = \frac{\sum_{fj} PVA_{fj,j,z,t} \cdot VA_{fj,j,z,t}}{VAC_{j,z,t}}$$

87. $$PVA_{'SOC',z,t} = \frac{W_{'TRA.EMPL',t} \cdot LD_{'TRA.EMPL',j,z,t} + R_{'KSOC',j,z,t} \cdot KD_{'KSOC',j,z,t}}{VA_{'SOC',z,t}}$$

88. $$PVA_{'IND',z,t} = \frac{W_{'TRA.AUTO',t} \cdot LD_{'TRA.AUTO',j,z,t} + R_{'KIND',j,z,t} \cdot KD_{'KIND',j,z,t}}{VA_{'IND',z,t}}$$

Trade

89. $$P_{i,z,t} = PT_{j,z,t}$$ (*dans le cas où il y a un seul output*)

90. $$P_{i,z,t} = \frac{\sum_{row} PE_{i,row,z,t} \cdot EX_{i,row,z,t} + PL_{i,z,t} \cdot DS_{i,z,t}}{XSI_{i,z,t}}$$

91. $$PPQ_{i,z,zj,t} = PL_{i,z,t}$$ (*dans le cas où le produit n'est vendu que sur une seule destination*)

92. $$PE^{FOB}_{i,row,z,t} = \left(PE_{i,row,z,t} + \sum_{ij} PCOMP_{ij,z,t} \cdot tmrg^{X}_{ij,i,row,z} \right) \cdot \left(1 + ttix_{i,row,z,t}\right)$$

93. $$PE^{FOBR}_{i,z,t} = e_{'RDM',t} \cdot PWM_{i,'RDM',z,t} + \sum_{ij} PCOMP_{ij,z,t} \cdot tmrg^{XR}_{ij,i,z}$$

94. $PPQ_{i,z,zj,t} = PD_{i,z,t}$ (*dans le cas où le produit n'est acheté que d'une seule région*)

95. $$PM_{i,row,z,t} = \left(1 + \sum_{gvt} ttim_{gvt,i,row,z,t}\right) \cdot e_{row,t} \cdot PWM_{i,row,z,t}$$

96. $$PCOMP_{i,z,t} = \frac{\sum_{row} PM_{row,i,z,t} \cdot IM_{row,i,z,t} + PL_{i,z,t} \cdot DD_{i,z,t}}{Q_{i,z,t}}$$

97. $$PDI_{i,z,t} = \left(1 + \sum_{gvt} ttic^{DI}_{gvt,i,z,t}\right) \cdot \left(PCOMP_{i,z,t} + \sum_{ij} PCOMP_{ij,z,t} \cdot tmrg^{DI}_{ij,i,z} \right)$$

98. $$PDF_{i,z,t} = \left(1 + ttic^{DF}_{i,z,t}\right) \cdot \left(PCOMP_{i,z,t} + \sum_{ij} PCOMP_{ij,z,t} \cdot tmrg^{DF}_{ij,i,z} \right)$$

Price indexes

99. $$PIXGDP_{z,t} = \sqrt{\frac{\sum_{j} \left(PVAC_{j,z,t} + \frac{\sum_{gvt} TIP_{gvt,j,z,t}}{VAC_{j,z,t}} \right) \cdot VAC^{O}_{j,z}}{\sum_{j} \left(PVAC^{O}_{j,z} \cdot VA^{O}_{j} + \sum_{gvt} TIP^{O}_{gvt,j,z} \right)} \frac{\sum_{j} \left(PVAC_{j,z,t} \cdot VAC_{j,z,t} + \sum_{gvt} TIP_{gvt,j,z,t} \right)}{\sum_{j} \left(PVA^{O}_{j,z} + \frac{\sum_{gvt} TIP^{O}_{gvt,j,z}}{VAC^{O}_{j,z}} \right) \cdot VAC_{j,z,t}}}$$

100. $$PIXGDP_{zs,t} = \sqrt{\frac{\sum_{j,z} \left(PVAC_{j,z,t} + \frac{\sum_{gvt} TIP_{gvt,j,z,t}}{VAC_{j,z,t}} \right) \cdot VAC^{O}_{j,z}}{\sum_{j,z} \left(PVAC^{O}_{j,z} \cdot VA^{O}_{j} + \sum_{gvt} TIP^{O}_{gvt,j,z} \right)} \frac{\sum_{j,z} \left(PVAC_{j,z,t} \cdot VAC_{j,z,t} + \sum_{gvt} TIP_{gvt,j,z,t} \right)}{\sum_{j,z} \left(PVA^{O}_{j,z} + \frac{\sum_{gvt} TIP^{O}_{gvt,j,z}}{VAC^{O}_{j,z}} \right) \cdot VAC_{j,z,t}}}$$

101. $$PIXCON_{z,t} = \frac{\sum_{i} PDF_{i,z,t} \cdot \sum_{h} C^{O}_{i,h,z}}{\sum_{ij} PDF^{O}_{ij,z} \cdot \sum_{h} C^{O}_{ij,h,z}}$$

102. $$PIXCON_{zs,t} = \frac{\sum_{i,z} PDF_{i,z,t} \cdot \sum_{h} C^{O}_{i,h,z}}{\sum_{ij,z} PDF^{O}_{ij,z} \cdot \sum_{h} C^{O}_{ij,h,z}}$$

103. $$PIXGVT_{gvt,z,t} = \prod_{i} \left(\frac{PDF_{i,z,t}}{PDF^{O}_{i,z}} \right)^{\gamma^{GVT}_{gvt,i,z}}$$

A1.1.6 EQUILIBRIUM

104. $$Q_{i,z,t} + VSTK^{OUT}_{i,z,t} + MUSA_{i,z,t} = DF_{i,z,t} + DIT_{i,z,t} + MRGN_{i,z,t}$$

105. $$\sum_{z} LS_{l,z,t} = \sum_{j,z} LD_{l,j,z,t} + LD^{RDC}_{l,t}$$

106. $$KS_{k,z,t} = \sum_{j} KD_{k,j,z,t}$$

107. $$IT_{t} = \sum_{h,z} SH_{h,z,t} + \sum_{f} SF_{f,t} + \sum_{z} SG_{'LOC',z,t} + \sum_{gvtsup} SG_{gvtsup,zs,t} + \sum_{row} SROW_{row,t}$$

108. $$IT^{PRI}_{t} = IT_{t} - IT^{PUB}_{t} - \sum_{i,z} PDF_{i,z,t} \cdot VSTK^{IN}_{i,z,t} + \sum_{i,z} PCOMP_{i,z,t} \left(VSTK^{OUT}_{i,z,t} + MUSA_{i,z,t} \right)$$

109. $$OPQ_{i,z,zj,t} = DPQ_{i,z,zj,t}$$

110. $$EX_{i,row,z,t} = EXD_{i,row,z,t}$$

A1.1.7 DYNAMIC EQUATIONS

111. $$KD_{k,j,z,t+1} = KD_{k,j,z,t} \left(1 - \delta_{k,j,z}\right) + IND_{k,j,z,t};$$
$$KD^{R}_{pub,z,t+1} = KD^{R}_{pub,z,t} \left(1 - \delta^{R}_{pub,z}\right) + IND^{R}_{pub,z,t}$$

112. $$IT^{PUB}_{t} = \sum_{a,pub,z} PIT^{A}_{a,z,t} \left(DINV_{a,pub,z,t} + DINV^{R}_{a,pub,z,t} \right)$$

113. $$IT^{PRI}_{t} = \sum_{a,bus,z} PIT^{A}_{a,z,t} \cdot DINV_{a,bus,z,t}$$

114. $$PIT^{A}_{a,z,t} = \frac{1}{A^{K}_{a,z}} \prod_{i} \left(\frac{PDF_{i,z,t}}{\gamma^{INV}_{i,a,z}} \right)^{\gamma^{INV}_{i,a,z}}$$

115. $$PK_{j,z,t} = \sum_{a} \gamma^{IND}_{a,j,z} \cdot PIT^{A}_{a,z,t}$$

116. $$IND_{k,bus,z,t} = \Phi_t \cdot \phi_{k,bus,z} \left[\frac{R_{k,bus,z,t}}{U_{k,bus,z,t}} \right]^{\sigma^{INV}_{k,bus}} KD_{k,bus,z,t}$$

117. $$IR_t = \frac{\sum_{k,j,z} \left(R_{k,j,z,t} - \delta_{k,j,z} \cdot PK_{j,z,t} \right) \cdot KD_{k,j,z,t}}{\sum_{k,j,z} PK_{j,z,t} \cdot KD_{k,j,z,t}}$$

118. $$U_{k,j,z,t} = PK_{j,z,t} \left(\delta_{k,j,z} + IR_t \right)$$

119. $$DINV_{a,j,z,t} = \gamma^{IND}_{a,j,z} \cdot \sum_{k} IND_{k,j,z,t}$$

120. $$IT^{A}_{a,z,t} = PIT^{A}_{a,z,t} \cdot \sum_{j} DINV_{a,j,z,t} + PIT^{A}_{a,z,t} \sum_{pub} DINV^{R}_{a,pub,z,t}$$

A1.1.8 GROSS DOMESTIC PRODUCT

121. $$GDP^{BP}_{z,t} = \sum_{j} PVAC_{j,z,t} \cdot VAC_{j,z,t} + \sum_{gvt} TIPT_{gvt,z,t}$$

122. $$GDP^{BP}_{zs,t} = \sum_{z} GDP^{BP}_{z,t}$$

123. $$GDP^{MP}_{z,t} = GDP^{BP}_{z,t} + \sum_{gvt} TPRCTS_{gvt,z,t}$$

124. $$GDP^{MP}_{zs,t} = \sum_{z} GDP^{MP}_{z,t}$$

125. $$GDP^{IB}_{z,t} = \sum_{l,j} W_{l,t} \cdot LD_{l,j,z,t} + \sum_{k,j} R_{k,j,z,t} \cdot KD_{k,j,z,t} + \sum_{gvt} \left[TIPT_{gvt,z,t} + TPRCTS_{gvt,z,t} \right]$$

126. $$GDP^{IB}_{zs,t} = \sum_{z} GDP^{IB}_{z,t}$$

127.
$$\begin{aligned} GDP^{FD}_{z,t} = & \sum_{i} PDF_{i,z,t} \cdot DF_{i,z,t} - \sum_{i} PCOMP_{i,z,t} \cdot \left(VSTK^{OUT}_{i,z,t} + MUSA_{i,z,t} \right) \\ & + \sum_{i,row} PE^{FOB}_{i,row,z,t} \cdot EXD_{i,row,z,t} + \sum_{i} PE^{FOBR}_{i,z,t} \cdot REX_{i,z,t} + \sum_{i,zj} PPQ_{i,z,zj,t} \cdot OPQ_{i,z,zj,t} \\ & - \sum_{i,row} e_{row,t} \cdot PWM_{i,row,z,t} \cdot IM_{i,row,z,t} - \sum_{i} e_{'RDM',t} \cdot PWM_{i,'RDM',z,t} \cdot REX_{i,z,t} \\ & - \sum_{i,zj} PPQ_{i,zj,z,t} \cdot DPQ_{i,zj,z,t} \end{aligned}$$

128. $GDP^{FD}_{zs,t} = \sum_{z} GDP^{FD}_{z,t}$

A1.1.9 REAL VARIABLES COMPUTED FROM PRICE INDEXES

129. $CTH^{REAL}_{h,z,t} = \frac{CTH_{h,z,t}}{PIXCON_{z,t}}$

130. $CTH^{REAL}_{h,zs,t} = \frac{\sum_{z} CTH_{h,z,t}}{PIXCON_{zs,t}}$

131. $G^{REAL}_{gvt,z,t} = \frac{G_{gvt,z,t}}{PIXGVT_{gvt,z,t}}$

132. $GDP^{BP_REAL}_{zt,t} = \frac{GDP^{BP}_{zt,t}}{PIXGDP_{zt,t}}$

133. $GDP^{MP_REAL}_{zt,t} = \frac{GDP^{MP}_{zt,t}}{PIXCON_{zt,t}}$

A1.2 Sets[18]

A1.2.1 INDUSTRIES AND PRODUCTS

All industries: $j, jj \in J = \{J_1, ..., J_{44}\}$

All products: $i, ij \in I = \{I_1, ..., I_{63}\}$

Public sector: $pub \in PUB \subset J = \{PUB_1, ..., PUB_5\}$

Private sector: $bus \in BUS \subset J = \{BUS_1, ..., BUS_{39}\}; BUS \cap PUB = \emptyset$

A1.2.2 PRODUCTION FACTORS

Categories of labor: $l \in L = \{TRA_EMPL, TRA_AUTO\}$

Categories of capital: $k \in K = \{KSOC, KIND\}$

A1.2.3 AGENTS

All agents: $ag, agj \in AG = H \cup F \cup GVT \cup ROW = \{MEN, SOC, ISBL, FED, PROV, LOC, RRQ, RPC, RDM, RDC\}$

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[18] See Appendix t for a complete description of set composition.

Households: $h, hj \in H \subset AG = \{MEN\}$

Firms: $f, fj \in F \subset AG = \{SOC, ISBL\}$

Governments: $gvt, gvtj \in GVT \subset AG = \{FED, PROV, LOC, RRQ, RPC\}$

High-level governments: $gvtsup \subset GVT = \{FED, PROV, RRQ, RPC\}$

Exterior agents: $row, rowj \in ROW \subset AG = \{RDM, RDC\}$

Non-government agents: $agng \in AGNG \subset AG = H \cup F \cup ROW = \{MEN, ENT, ISBL, RDM, RDC\}$

Domestic agents: $agd \in AGD \subset AG = H \cup F \cup GVT = \{MEN, ENT, ISBL, FED, PROV, LOC, RRQ, RPC\}$

A1.2.4 ASSET CATEGORIES

Assets: $a, aj \in A = \begin{Bmatrix} BRES, BNRES_PRI, GENIE_PRI, MM_PRI, PI_PRI, \\ BNRES_PUB, GENI_PUB, MM_PUB, PI_PUB, INF_ROUT \end{Bmatrix}$

A1.2.5 LEGAL FORMS OF ENTREPRISE

Legal forms of business organization: $fj, fjj \in FJ = \{SOC, IND\}$

A1.2.6 REGIONS

All regions and supraregional accounts: $zt, ztj \in ZT = \{RANA-01, ..., RANA-16, SUPRA\}$

Regions only: $z, zj \in Z \subset ZT = \{RANA-01, ..., RANA-16\}$

Supraregional accounts only: $zs, zsj \in ZS \subset ZT = \{SUPRA\}$

A1.2.7 PERIODS

Periods: $t \in T = \{2011 \text{ à } 2025\}$

A1.3 Endogenous variables

A1.3.1 VOLUME VARIABLES

$C_{i,h,z,t}$: Consumption of product *i* by household *h*

$CF_{i,f,z,t}$: Consumption of product *i* by firm type *f* (volume)

$CG_{i,gvt,z,t}$: Public consumption of product *i* by government agent *gvt* (volume)

$CI_{j,z,t}$: Intermediate consumption of industry *j*

$CTH^{REAL}_{h,zt,t}$: Real consumption of household *h*

$DD_{i,z,t}$: Domestic demand for product *i* produced locally

$DF_{i,z,t}$: Final demand for product *i*

$DI_{i,j,z,t}$: Intermediate consumption of product *i* by industry *j*

$DINV_{a,j,z,t}$: Volume of investment in asset *a* for industry *j*

$DIT_{i,z,t}$: Intermediate consumption of product *i*

$DPQ_{i,zj,z,t}$: Demand of region *z* for product *i* originating from region *zj*

$DS_{i,z,t}$: Total supply of product *i* on the Quebec market

$EX_{i,row,z,t}$: Volume of product *i* exported to trade partner *row*

$EXD_{i,row,z,t}$: Demand by trade partner *row* for exports of product *i*

$GDP^{BP_REAL}_{zt,t}$: Real GDP at basic prices

$GDP^{MP_REAL}_{zt,t}$: Real GDP at market prices

$IM_{i,row,z,t}$: Volume of imported product *i*

$IND_{k,bus,z,t}$: Volume of investment in new type *k* capital for industry *bus*

$INV_{i,a,z,t}$: Final demand of product *i* for private investment purposes in asset *a*

$INVT_{i,z,t}$: Final demand of product *i* for purposes of public investment

$KS_{k,z,t}$: Supply of type *k* capital

$LD_{l,j,z,t}$: Demand for type *l* labor by industry *j*

$MRGN_{i,z,t}$: Demand for product *i* as margin

$OPQ_{i,z,zj,t}$: Supply of product *i* by region *z* to region *zj*

$Q_{i,z,t}$: Total volume of demand for composite product *i*

$REX_{i,z,t}$: Volume of re-exports of product *i*

$VA_{fj,j,z,t}$: Value added created by businesses of legal form *fj* in industry *j*

$VAC_{j,z,t}$: Value added of industry *j*

$XS_{j,i,z,t}$: Output of product *i* by industry *j*

$XSI_{i,z,t}$: Total output of product *i*

$XST_{j,z,t}$: Aggregate output of industry *j*

A1.3.2 PRICE VARIABLES

$e_{RdC,t}$: Price ratio between Quebec and the RoC (overall level of prices of products traded with the RoC relative to the general level of Quebec prices)

IR_t : Interest rate

$P_{i,z,t}$: Basic price of product *i*

$PCOMP_{i,z,t}$: Basic price of composite product *i* (includes only tariffs)

$PD_{i,z,t}$: Composite price of product *i* purchased in Quebec

$PDF_{i,z,t}$: Market price of composite product *i* (final demand)

$PDI_{i,z,t}$: Market price of composite product *i* (intermediate demand)

$PCI_{j,z,t}$: Index of intermediate consumption prices of industry *j*

$PE_{i,row,z,t}$: Price received for product *i* exported to *row* (excludes margins and export taxes)

$PE^{FOB}_{i,row,z,t}$: FOB price of product *i* exported to *row* (in local currency)

$PE^{FOBR}_{i,z,t}$: FOB price of re-exported product *i* réexporté (in local currency)

$PIT^{A}_{a,z,t}$: Price of asset *a*

$PIXCON_{zt,t}$: Consumer price index

$PIXGDP_{zt,t}$: GDP deflator

$PIXGVT_{gvt,z,t}$: Price index of public expenditures by government

$PK_{j,z,t}$: Price of new capital for industry *j*

$PL_{i,z,t}$: Price of product *i* sold in Quebec

$PM_{i,row,z,t}$: Price of product *i* imported from *row* (including tariffs)

$PP_{j,z,t}$: Unit cost of industry *j* excluding taxes and subsidies on production

$PPQ_{i,z,zj,t}$: Price of product *i* sold by region *z* to region *zj*

$PT_{j,z,t}$: Basic price of industry *j*'s output

$PVA_{fj,j,z,t}$: Price of value added generated by businesses of legal form *fj* in industry *j*

$PVA_{j,z,t}$: Price of the composite value added of industry *j*

$R_{k,j,z,t}$: Rental rate of type *k* capital paid by industry *j*

$U_{k,j,z,t}$: User cost of type *k* capital for industry *j*

$W_{l,t}$: Wage rate of type *l* labor

A1.3.3 NOMINAL VALUE VARIABLES

$CAB_{row,t}$: Current account balance

$CTH_{h,z,t}$: Consumption budget of household *h*

$G_{gvt,z,t}$: Current expenditures on goods and services of government *gvt*

$GDP^{BP}_{zt,t}$: GDP at basic prices

$GDP^{FD}_{zt,t}$: GDP at market prices (computed from final demand)

$GDP^{IB}_{zt,t}$: GDP at market prices (computed from incomes)

$GDP^{MP}_{zt,t}$: GDP at market prices

IT_t : Total investment expenditures

$IT^{A}_{a,z,t}$: Investment expenditures on asset *a*

$RP^{PAYE}_{ag,zt,t}$: Property income paid by agent *ag*

$RP^{RECU}_{ag,zt,t}$: Property income received by agent *ag*

$SF_{f,t}$: Savings of firm *f*

$SG_{gvt,zt,t}$: Savings by government *gvt*

$SH_{h,z,t}$: Savings by household *h*

$SROW_{row,t}$: Savings by external agent *row*

$TDF_{gvt,f,t}$: Direct taxes paid by firm *f* to government *gvt*

$TDFT_{gvt,t}$: Total revenue of government *gvt* from direct taxes on business

$TDH_{gvt,h,z,t}$: Direct taxes paid by household *h* to government *gvt*

$TDHT_{gvt,z,t}$: Total revenue of government *gvt* from direct taxes on households

$TIC_{gvt,i,z,t}$: Indirect taxes on product *i* collected by government *gvt*

$TICT_{gvt,z,t}$: Total revenue of government *gvt* from indirect taxes on products, net of subsidies

$TIM_{gvt,i,z,t}$: Indirect import taxes on product *i* collected by government *gvt*

$TIMT_{gvt,z,t}$: Total revenue of government *gvt* from indirect taxes on imports

$TIP_{gvt,j,z,t}$: Indirect taxes on the production of industry *j* collected by government *gvt*

$TIPT_{gvt,z,t}$: Total revenue of government *gvt* from indirect taxes on production

$TIS_{gvt,j,z,t}$: Cost of subsidy on the composite output of industry *j*

$TIX_{gvt,i,z,t}$: Revenue of government *gvt* from export taxes on product *i*

$TIXT_{gvt,z,t}$: Total revenue of government *gvt* from export taxes

$TPRCTS_{gvt,z,t}$: Total revenue of government *gvt* from indirect taxes on products

$TR_{ag,zt,agj,ztj,t}$: Transfers from agent *agj* in zone *ztj* to agent *ag* in zone *zt*

$YDF_{f,t}$: Disposable income of firm *f*

$YDH_{h,z,t}$: Disposable income of household *h*

$YF_{f,t}$: Total income of firm *f*

$YFK_{f,t}$: Capital income of firm *f*

$YFTR_{f,t}$: Transfer income of firm *f*

$YG_{gvt,zt,t}$: Total income of government *gvt*

$YGK_{gvt,z,t}$: Capital income of government *gvt*

$YGTR_{gvt,zt,t}$: Transfer income of government*gvt*

$YH_{h,z,t}$: Total income of household *h*

$YHK_{h,z,t}$: Capital income of household *h*

$YHL_{h,t}$: Labor income of household *h*

$YHTR_{h,z,t}$: Transfer income of household *h*

$YROW_{row,t}$: Income of exterior agent *row*

A1.4 Exogenous variables

$C^{MIN}_{i,h,z,t}$: Minumum consumption of product *i* by household *h*

$CTF_{f,t}$: Final consumption expenditures of firms *f* (NPISH)

$e_{RDM,t}$: Exchange rate (price of foreign currency in terms of local currency)

$G^{REAL}_{gvt,z,t}$: Real public expenditures of government *gvt*

$IND_{k,pub,z,t}$: Volume of investment in new type *k* capital for public industry *pub*

$IND^{R}_{a,pub,z,t}$: Volume of investment in road infrastructure

$KD_{k,j,z,t}$: Demand for type *k* capital by industry *j* (determined from preceding period)

$KD^{R}_{pub,z,t}$: Stock of road infrastructure by level of government (determined from preceding period)

$LD^{RDC}_{l,t}$: Demand for type *l* labor by the rest of Canada

$LS_{l,z,t}$: Supply of type *l* labor from region *z*

$PWM_{i,row,t}$: World price of product *i* imported from *row*

$PWX_{i,row,z,t}$: World price of product *i* exported to *row*

$PWXR_{i,z,t}$: World price of re-exported product *i*

$sh0_{h,z,t}$: Intercept (savings function of household *h*)

$sh1_{h,z,t}$: Slope (savings function of household *h*)

$tr0_{gvt,h,t}$: Intercept (transfers from household *h* to government *gvt*)

$tr1_{gvt,h,z,t}$: Marginal rate of transfers from household *h* to government *gvt*

$ttdf0_{gvt,f,t}$: Intercept (direct taxes paid by firm *f* to government *gvt*)

$ttdf1_{gvt,f,t}$: Marginal tax rate of firm *f*

$ttdh0_{gvt,h,z,t}$: Intercept (direct taxes paid by household *h* to government *gvt*)

$ttdh1_{gvt,h,z,t}$: Marginal tax rage of household *h*

$ttic^{DF}_{gvt,i,z,t}$: Tax rate on product *i* (final demand)

$ttic^{DI}_{gvt,i,z,t}$: Tax rate on product *i* (intermediate demand)

$ttim_{gvt,i,row,z,t}$: Tariff rate on product *i* imported from *row*

$ttip_{gvt,j,z,t}$: Tax rate on the production of industry *j*

$ttis_{gvt,j,z,t}$: Subsidy rate on the composite output of industry *j*

$ttix_{gvt,i,row,z,t}$: Export tax rate on product *i* exported to *row*

$VSTK^{IN}_{i,z,t}$: Inventory additions of product *i*

$VSTK^{OUT}_{i,z,t}$: Inventory withdrawals of product *i*

$W^{RDC}_{l,t}$: Wage rate paid by the rest of Canada for type *l* labor

$MUSA_{i,z,t}$: Used goods *i*

A1.5 Parameters

$A^{K}_{a,z}$: Scale parameter (price of asset *a*)

$aij_{i,j,z}$: Input-output coefficient

$B^{M}_{i,z}$: Scale parameter (CES – composite product)

$B^{MQ}_{i,z}$: Scale parameter (CES – imports by region)

$B^{VA}_{fj,j,z}$: Scale parameter (CES – value added)

$B^{VAC}_{j,z}$: Scale parameter (CES – composite value added)

$B^{X}_{i,z}$: Scale parameter (CET – Quebec exports and domestic sales)

$B^{XQ}_{i,z}$: Scale parameter (CET – regional exports)

$B^{XT}_{j,z}$: Scale parameter (CET – total output)

$\beta^{MQ}_{i,z,zj}$: Distribution parameter (CES – imports by region)

$\beta^{M.RDC}_{i,z}$: Distribution parameter (CES – composite product)

$\beta^{M.RDM}_{i,z}$: Distribution parameter (CES – composite product)

$\beta^{VA}_{fj,j,z}$: Distribution parameter (CES – value added)

$\beta^{VAC}_{j,z}$: Distribution parameter (CES – composite value added)

$\beta^{XQ}_{i,z,zj}$: Distribution parameter (CET – exports to regions)

$\beta^{X.RDC}_{i,z}$: Distribution parameter (CET – Quebec exports and domestic sales)

$\beta^{X.RDM}_{i,z}$: Distribution parameter (CET – Quebec exports and domestic sales)

$\beta^{XT}_{j,i,z}$: Distribution parameter (CET – total output)

$\delta_{k,j,z}$: Rate of depreciation of type *k* capital in industry *j*

$\delta^{R}_{pub,z}$: Rate of depreciation of road infrastructures

η : Price elasticity of parameters and indexed transfers

$\gamma^{F}_{i,f,z}$: Part du produit *i* dans la consommation finale de l'entreprise *f*

$\gamma^{GVT}_{i,gvt,z}$: Share of product *i*in public expenditures by government *gvt*

$\gamma_{a,j,z}^{IND}$: Share of asset *a* in investment for industry *j* (Leontief)

$\gamma_{i,a,z}^{INV}$: Share of product *i* in investment expenditures on asset *a*

$\gamma_{i,h,z}^{LES}$: Marginal share of product *i* in household *h*'s consumption budget

$io_{j,z}$: Coefficient (Leontief – intermediate consumption)

$\lambda_{h,z}^{RPP}$: Ratio of property income paid by household *h* over total consumption expenditures

$\lambda_{f,zs}^{RPP}$: Fraction of firm *f* total income paid in property income

$\lambda_{ag,zt}^{RPR}$: Share of property income received by agent *ag*

$\lambda_{ag,k,zt}^{RK}$: Share of type *k* capital income received by agent *ag*

$\lambda_{ag,zt,agj,ztj}^{TR}$: Distribution parameter (transfer functions)

$\lambda_{h,l,z}^{WL}$: Share of type *l* labor income received by household *h*

$n_{z,t}$: Population growth rate in region *z*

$\phi_{k,j,z}$: Scale parameter (allocation of investment among industries)

pop_t : Population index

ρ_i^M : Elasticity parameter (CES – composite product); $-1 < \rho_i^M < \infty$

$\rho_{i,z}^{MQ}$: Elasticity parameter (CES – imports by region); $-1 < \rho_i^M < \infty$

$\rho_{fj,j}^{VA}$: Elasticity parameter (CES – value added); $1 < \rho_j^{VA} < \infty$

ρ_j^{VAC} : Elasticity parameter (CES – composite value added) ; $1 < \rho_j^{VA} < \infty$

ρ_i^X : Elasticity parameter (CET – Quebec exports and domestic sales); $1 < \rho_{j,i}^X < \infty$

$\rho_{i,z}^{XQ}$: Elasticity parameter (CET – export to regions); $1 < \rho_{j,i}^X < \infty$

ρ_j^{XT} : Elasticity parameter (CET – total output); $1 < \rho_j^{XT} < \infty$

$\sigma_{k,bus}^{INV}$: Elasticity of investment demand relative to Tobin's *q*

σ_i^M : Elasticity of substitution (CES – composite product); $0 < \sigma_i^M < \infty$

$\sigma_{i,z}^{MQ}$: Elasticity of substitution (CES –imports by region); $0 < \sigma_i^M < \infty$

$\sigma_{fj,j}^{VA}$: Elasticity of transformation (CES – value added) ; $0 < \sigma_j^{VA} < \infty$

σ_j^{VAC} : Elasticity of transformation (CES – composite value added) ; $0 < \sigma_j^{VA} < \infty$

σ_i^X : Elasticity of transformation (CET – Quebec exports and domestic sales); $0 < \sigma_{j,i}^X < \infty$

$\sigma_{i,row}^{XD}$: Price-elasticity of demand for exports of product *i* by *row*

σ_i^{REX} : Price-elasticity of external demand for re-exported product *i*

$\sigma_{i,z}^{XQ}$: Elasticity of transformation (CET – exports to regoins); $0 < \sigma_{j,i}^X < \infty$

σ_j^{XT} : Elasticity of transformation (CET – total output); $0 < \sigma_j^{XT} < \infty$

$tmrg^{DF}_{i,ij,z}$: Rate of margin *i* applied to product *ij* (final demand)

$tmrg^{DIT}_{i,ij,z}$: Rate of margin *i* applied to product *ij* (intermediate demand)

$tmrg^{X}_{i,ij,row,z}$: Rate of margin *i* applied to product *ij* (exports)

$tmrg^{XR}_{i,ij,z}$: Rate of margin *i* applied to product *ij* (re-exports)

$v_{j,z}$: Coefficient (Leontief – value added)

APPENDIX 2 – SAM ACCOUNTS AND CLASSIFICATIONS

A2.1 INDUSTRIES

	Industry name	North American Industry Classification System 2007 definition	Input-Output Industry Codes
01-AGRI	Crop and animal production	11A: 111, 112	BS11A
02-FORET	Forestry and logging	113	BS113
03-CHASSE	Fishing, hunting and trapping	114	BS114
04-STAGR	Support activities for agriculture and forestry	115	BS115
05-MINES	Mining, quarrying, and oil and gas extraction	21	BS210
06-SER_PUB	Utilities	22	BS220
07-CONSTRU	Construction	23	BS23A à BS23E
08-ALIM	Food manufacturing	311	BS31110, BS31130 à BS31170, BS311A0
09-TABAC_BOI	Beverage and tobacco product manufacturing	312	BS31211, BS31212, BS2121A, BS31220
10-TEXT	Textile and textile product mills	31A: 313, 314	BS31A00
11-VETEM	Clothing and leather and allied product manufacturing	31B: 315, 316	BS31B00
12-BOIS	Wood product manufacturing	321	BS32100
13-PAPIER	Paper manufacturing	322	BS32210, BS32220
14-IMPRESSION	Printing and related support activities	323	BS32300
15-RAFFIN	Petroleum and coal product manufacturing	324	BS32400
16-CHIMIE	Chemical manufacturing	325	BS32510 à BS32540, BS325A0
17-PLAST	Plastics and rubber products manufacturing	326	BS32610, BS32620
18-MIN_NMET	Non-metallic mineral product manufacturing	327	BS32730, BS327A0
19-METAL_PREM	Primary metal manufacturing	331	BS33100
20-METAL_PROD	Fabricated metal product manufacturing	332	BS33200
21-MACHINES	Machinery manufacturing	333	BS33300
22-ELECTRON	Computer and electronic product manufacturing	334	BS33410, BS334B0
23-APP_ELEC	Electrical equipment, appliance and component manufacturing	335	BS33520, BS335A0
24-MAT_TRANSP	Transportation equipment manufacturing	336	BS33610 à BS33690

25-MEUBLES	Furniture and related product manufacturing	337	BS33700
26-AUT_FAB	Miscellaneous manufacturing	339	BS33900
27-GROS	Wholesale trade	41	BS410
28-DETAIL	Retail trade	4A: 44 à 45	BS4A0
29-TRANSPORT	Transportation and warehousing	4B: 48 à 49	BS4B0
30-INFO_CULT	Information and cultural industries	51	BS510
31-FIN_IMM	Finan.&insur., real estate & rental & leasing, management of companies and enterp.	52, 53 sauf 5311A	BS52B00, BS522A00, BS52410, BS524200, BS52A000, BS53110, BS531A00, BS53B00
32-PROPRIO	Owner-occupied dwellings	5311A	BS5311A
33-SER_PROF	Professional, scientific and technical services	54	BS540
34-SER_ADMIN	Administrative and support, waste management and remediation services	55, 56	BS551113, BS560
35-SER_ENSEIG	Educational services	61 sauf GS610	BS610, NP61000
36-SANTE_AS	Health care and social assistance	62 sauf GS620	BS620, NP621000, NP62400
37-ARTS	Arts, entertainment and recreation	71	BS710, NP71000
38-RESTO_HOTEL	Accommodation and food services	72	BS720
39-AUT_SERV	Other services (except public administration)	81	BS810, NP81310, NP813A00
40-G_ENSEIGN	Public educational services		GS610
41-G_SANTE	Public health care and social assistance		GS620
42-G_AFED	Federal government public administration	911	GS911
43-G_APROV	Provincial and territorial public administration	912	GS912
44-G_AMUN	Local, municipal, regional and aboriginal public administration	91A: 913, 914	GS913, GS914

A2.2 PRODUCTS

Product	Title	Code
01-RECOLT	Grains and other crop products	M111B
02-ANIMAUX	Live animals	M112A
03-AUT_AGR	Other farm products	M11D0
04-FORET	Forestry products and services	M11E0
05-POISSON	Fish and seafood	M1140
06-STAGR	Support services related to farming and forestry	M1150
07-COMBUST	Mineral fuels	M21B0
08-MIN_METAL	Metal ores and concentrates	M2122
09-MIN_N_METAL	Non-metallic minerals	M2123
10-STMINES	Mineral support services	M2130
11-EXPLO_MINE	Mineral and oil and gas exploration	M21A0
12-SERV_PUB	Utilities	M2200
13-CSTR_RES	Residential construction	M23A0
14-CSTR_N_RES	Non-residential buildings	M23B0
15-CSTR_GENIE	Engineering construction	M23C0
16-CSTR_REPAR	Repair construction services	M23D0
17-ALIM_BOIS	Food and non-alcoholic beverages	M31C0
18-TABAC_ALCOOL	Alcoholic beverages and tobacco products	M312A
19-TEXT_VET	Textile products	M31D0
20-BOIS	Wood products	M3210
21-PAPIER	Wood pulp	M3220
22-IMPRESSION	Printed products and services	M3230
23-PET_RAFF	Refined petroleum products (except petrochemicals)	M3240
24-CHIMIE	Chemical products	M3250
25-PLAST	Plastic and rubber products	M3260
26-MIN_NMET	Non-metallic mineral products	M3270
27-METAL_PREM	Primary metallic products	M3310
28-METAL_PROD	Fabricated metallic products	M3320
29-MACHINES	Industrial machinery	M3330
30-ELECTRON	Computer and electronic products	M334C
31-APP_ELEC	Electrical equipment	M3350
32-MAT_TRANSP	Transportation equipment	M336A
33-PIECES_VEH	Motor vehicle parts	M3363
34-MEUBLES	Furniture and related products	M3370
35-AUT_FAB	Other manufactured products and custom work	M3B00
36-GROS	Wholesale margins and commissions	M4100
37-DETAIL	Retail margins and commissions	M4A00.1[19]

[19] MPS4A0001+MPS4A0004

38-USAGE	Used goods	M4A00.2[20]
39-TRANSPORT	Transportation and related services	M4B00
40-INFO_CULT	Information and cultural services	M51D0
41-PUB_AUDIO	Published and recorded media products	M51E0
42-TELECOM	Telecommunications	M5170
43-INTER_FIN	Depository credit intermediation	M52C0
44-FIN_ASS	Other finance and insurance	M5F00
45-IMMOB	Real estate rental and leasing and rights to non-financial intangible assets	M53D0
46-LOY_IMP	Imputed rental of owner-occupied dwellings	M53C0
47-SER_PROF	Professional services (except software and research and development)	M541E
48-LOGICIELS	Software	M5E00
49-RetD	Research and development	M5417
50-SER_ADMIN	Admin.&support, head office, waste management & remediation serv.	M5G00
51-SER_ENSEIG	Education services	M6100
52-SANTE_AS	Health and social assistance services	M6200
53-ARTS	Arts entertainment and recreation services	M7100
54-RESTO_HOTEL	Accommodation and food services	M7200
55-AUT_SERV	Other services	M8100
56-VT_ISBL	Sales of other services by Non-Profit Institutions Serving Households	M9A00
57-VT_ADMPUB	Sales of other government services	M9B00
58-SERV_ISBL	Services provided by Non-Profit Institutions Serving Households	N0000
59-G_ENSEIGN	Education services provided by government sector	G6100
60-G_SANTE	Health services provided by government sector	G6200
61-G_AFED	Other federal government services	G9110
62-G_APROV	Other provincial and territorial government services	G9120
63-G_AMUN	Other municipal government services	G9130
64-G_AAUT	Other aboriginal government services	G9140

A2.3 PRODUCTION FACTORS

TRA_EMPL	Employees
TRA_AUTO	Self-employed workers
SUP_TRAV	Supplementary labor income
KIND	Capital of unincorporated businesses
KSOC	Capital of corporations

A2.4 AGENTS

MEN	Households
ISBL	Non-profit institutions serving households

[20] MPS4A0002+MPS4A0003

SOC	Corporations
RPROPRI	Property income
FED	Federal government
FED_TD	Federal income tax
TPS	GST
FED_TI	Other indirect federal taxes
PROV	Provincial government
PROV_TD	Provincial income tax
TVQ	TVQ
PROV_TI	Other indirect provincial taxes
LOC	Local governments
TIP	Taxes on production
SIP	Subsidies on production
SIC	Subsidies on products
RRQ	Régie des rentes du Québec
RPC	Canada Pension Plan
RdC	Rest of Canada
RdM	Rest of the world outside Canada

A2.5 LEGAL FORM OF ENTREPRISE

SOC	Corporations
IND	Unincorporated business

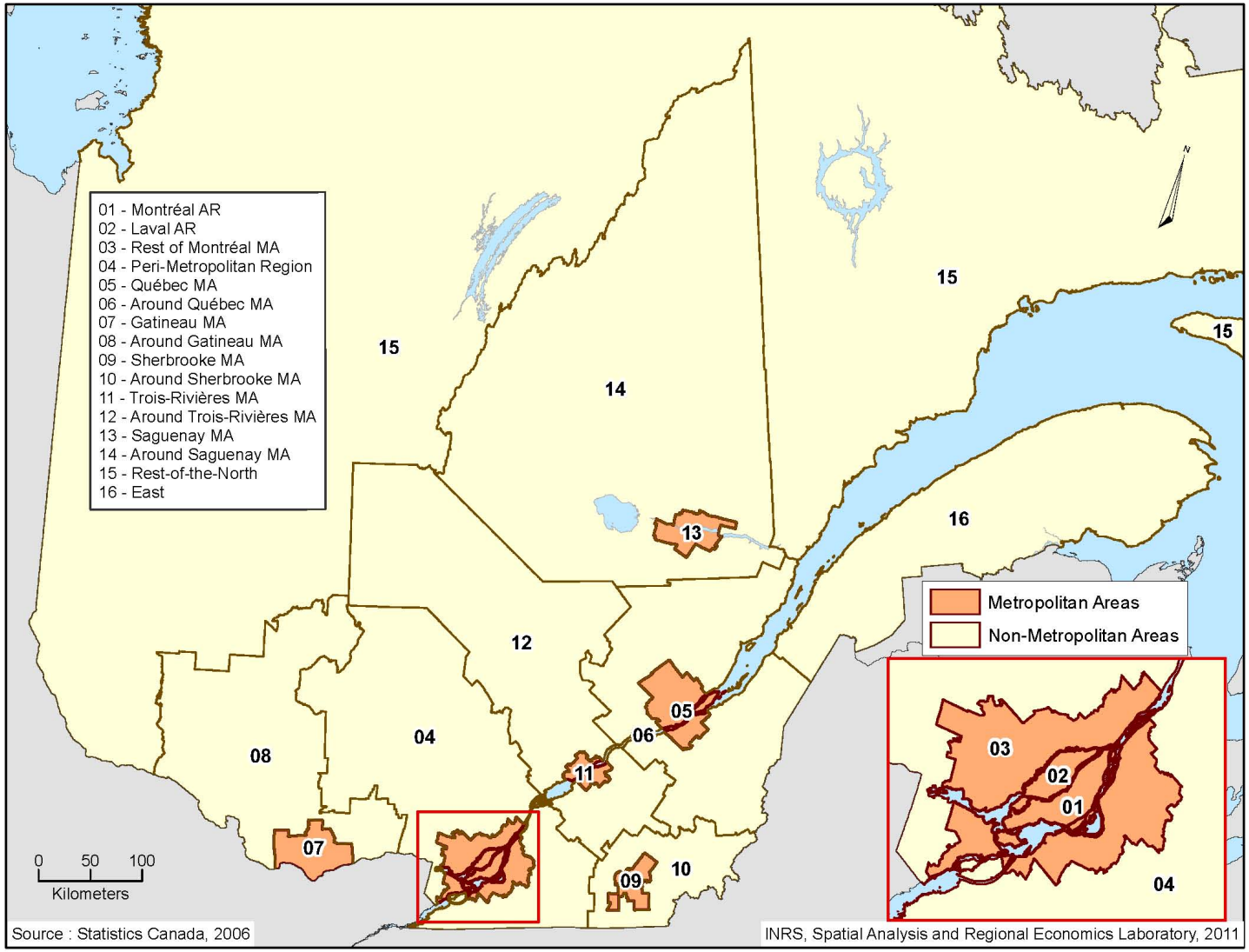
A2.6 ASSET CATEGORIES (CAPITAL)

BRES	Residential buildings
BNRES_PRI	Non-residential buildings (corporations and NPISHs)
GENIE_PRI	Engineering structures Ouvrages de génie (corporations and NPISHs)
MM_PRI	Machinery and equipment (corporations and NPISHs)
PI_PRI	Intellectual property products(corporations and NPISHs)
BNRES_PUB	Non-residential buildings (public administrations)
GENIE_PUB	Engineering structures (public administrations)
MM_PUB	Machinery and equipment (public administrations)
PI_PUB	Intellectual property products (public administrations)
INF_ROUT	Road infrastructures

APPENDIX 3 – GEOGRAPHY OF THE ANALYTICAL REGIONS (ANAR)

Code	Name	Composition
RANA-01	RA Montréal	RA Montréal
RANA-02	RA Laval	RA Laval
RANA-03	Reste RMR Montréal	RMR Montréal – [RA Montréal + RA Laval]
RANA-04	Péri-Montréal	[RA Laurentides + RA Lanaudière + RA Montérégie] – Reste RMR Montréal
RANA-05	RMR Québec	RMR Québec
RANA-06	Péri-Québec	[RA Chaudière-Appalaches + RA Capital-Nationale] – RMR Québec
RANA-07	RMR Gatineau	partie de la RMR Ottawa-Gatineau en territoire québécois
RANA-08	Péri-Gatineau	RA Outaouais – RMR Gatineau
RANA-09	RMR Sherbrooke	RMR Sherbrooke
RANA-10	Péri-Sherbrooke	RA Estrie – RMR Sherbrooke
RANA-11	RMR Trois-Rivières	RMR Trois-Rivières
RANA-12	Péri-Trois-Rivières	[RA Mauricie + RA Centre-du-Québec] – RMR Trois-Rivières
RANA-13	RMR Saguenay	RMR Saguenay
RANA-14	Péri-Saguenay	RA Saguenay—Lac-Saint-Jean – RMR Saguenay
RANA-15	Reste du Nord	RA Abitibi-Témiscamingue + RA Nord-du-Québec + RA Côte-Nord
RANA-16	Est	RA Bas-Saint-Laurent + RA Gaspésie—Îles-de-la-Madeleine
RdC	Reste-du-Canada	
RdM	Reste-du-monde	

Map 1 – Analytical regions



APPENDIX 4 – AGGREGATED SAM OF QUEBEC 2011 (G$)

	Labor	Capital	House-holds	Entre-prises	Govt.	RoW	Margins	Indus-tries	Products	Exports	Composite	Interm. demand	Final demand	Svng/ Inv.	TOTAL
Labor						4.5		177.9							**182.4**
Capital								126.8							**126.8**
Households	182.4	27.6		33.5	43.3	0.5									**287.3**
Entreprises		47.9	12.7		24.8										**85.4**
Governments		0.8	68.1	21.9	31.3	1.4		12.0		0.2	0.8	4.7	23.3		**164.6**
Rest of the world			0.9							6.0	173.7				**180.6**
Margins										9.2		19.5	36.8		**65.5**
Industries									606.4						**606.4**
Products										142.9	463.5				**606.4**
Exports						158.2									**158.2**
Composite							65.5					265.4	312.6		**643.6**
Intermediates								289.6							**289.6**
Final demand			200.2	5.8	83.7									83.0	**372.7**
Savings/Invest.		50.6	5.3	24.2	-18.5	15.9					5.5				**83.0**
TOTAL	**182.4**	**126.8**	**287.3**	**85.4**	**164.6**	**180.6**	**65.5**	**606.4**	**606.4**	**158.2**	**643.6**	**289.6**	**372.7**	**83.0**	

APPENDIX 5 – FREE MODEL PARAMETERS

Parameters relating to industries

	sigma_KD	sigma_LD	sigma_VA	sigma_XT
01-AGRI	1.536	1.536	0.768	0.4
02-FORET	1.536	1.536	0.768	0.4
03-CHASSE	1.536	1.536	0.768	0.4
04-STAGR	1.536	1.536	0.768	0.4
05-MINES	1.9	1.9	0.95	0.4
06-SER_PUB	0.6	0.6	0.3	0.4
07-CONSTRU	1.01	1.01	0.5	2
08-ALIM	2.2	2.2	1.1	0.8
09-TABAC_BOI	2.2	2.2	1.1	0.8
10-TEXT	2.2	2.2	1.1	0.8
11-VETEM	2.2	2.2	1.1	2
12-BOIS	1.622	1.622	0.811	0.8
13-PAPIER	2.2	2.2	1.1	0.8
14-IMPRESSION	2.2	2.2	1.1	0.8
15-RAFFIN	1.722	1.722	0.861	2
16-CHIMIE	2.2	2.2	1.1	2
17-PLAST	2.2	2.2	1.1	0.8
18-MIN_NMET	2.2	2.2	1.1	2
19-METAL_PREM	2.2	2.2	1.1	0.8
20-METAL_PROD	2.2	2.2	1.1	0.8
21-MACHINES	1.48	1.48	0.74	0.8
22-ELECTRON	1.48	1.48	0.74	2
23-APP_ELEC	1.48	1.48	0.74	2
24-MAT_TRANSP	1.734	1.734	0.867	2
25-MEUBLES	1.48	1.48	0.74	2
26-AUT_FAB	1.48	1.48	0.74	2
27-GROS	0.6	0.6	0.3	0.8
28-DETAIL	0.6	0.6	0.3	0.8
29-TRANSPORT	0.6	0.6	0.3	0.8
30-INFO_CULT	1.6	1.6	0.8	0.8
31-FIN_IMM	1.6	1.6	0.8	0.8
32-PROPRIO	1.6	1.6	0.8	0.8
33-SER_PROF	1.6	1.6	0.8	0.8
34-SER_ADMIN	1.6	1.6	0.8	0.8
35-SER_ENSEIG	1.6	1.6	0.8	0.8
36-SANTE_AS	1.6	1.6	0.8	0.8
37-ARTS	1.6	1.6	0.8	0.8
38-RESTO_HOTEL	1.6	1.6	0.8	2
39-AUT_SERV	1.6	1.6	0.8	0.8
40-G_ENSEIGN	1.6	1.6	0.8	0.8
41-G_SANTE	1.6	1.6	0.8	0.8
42-G_AFED	1.6	1.6	0.8	0.8
43-G_APROV	1.6	1.6	0.8	0.8
44-G_AMUN	1.6	1.6	0.8	0.8
45-G_AAUT	1.6	1.6	0.8	0.8

Parameters relating to products

	sigma_M	sigma_XD	sigma_X
01-RECOLT	1.5	0.8	3.786
02-ANIMAUX	4.5	0.8	3.786
03-AUT_AGR	0.9	0.8	3.786
04-FORET	1.5	0.8	3.786
05-POISSON	3.8	0.8	0.2
06-STAGR	1.5	0.8	3.786
07-COMBUST	1.5	0.8	0.892
08-MIN_METAL	10	0.8	1.05
09-MIN_N_METAL	1.2	0.8	1.05
10-STMINES	1.2	0.8	1.05
11-EXPLO_MINE	1.2	0.8	1.05
12-SERV_PUB	1.5	2	1.1
13-CSTR_RES	1.5	0.8	0.5
14-CSTR_N_RES	1.5	0.8	0.5
15-CSTR_GENIE	1.5	0.8	0.5
16-CSTR_REPAR	1.5	0.8	0.5
17-ALIM_BOIS	2	1.3	0.752
18-TABAC_ALCOOL	2	1.3	0.784
19-TEXT_VET	1.5	1.3	0.394
20-BOIS	2	1.3	0.541
21-PAPIER	1.8	1.3	0.425
22-IMPRESSION	1.8	1.8	0.425
23-PET_RAFF	1.8	1.3	0.892
24-CHIMIE	1.8	1.3	0.367
25-PLAST	2	1.3	0.276
26-MIN_NMET	2	1.3	0.216
27-METAL_PREM	1.5	1.6	0.424
28-METAL_PROD	1.5	1.6	0.499
29-MACHINES	0.8	1.6	0.379
30-ELECTRON	2	1.6	0.411
31-APP_ELEC	2	1.6	0.311
32-MAT_TRANSP	1.5	1.6	1.01

	sigma_M	sigma_XD	sigma_X
33-PIECES_VEH	0.982	1.6	1.01
34-MEUBLES	0.55	1.3	0.541
35-AUT_FAB	2	1.3	0.411
36-GROS	1.5	1.4	1.1
37-DETAIL	1.5	1.4	1.1
38-USAGE	1.5	1.4	1.1
39-TRANSPORT	1.5	1.4	1.1
40-INFO_CULT	1.5	1.4	1.1
41-PUB_AUDIO	1.5	1.4	1.1
42-TELECOM	1.5	1.4	1.1
43-INTER_FIN	1.5	1.4	1.1
44-FIN_ASS	1.5	1.4	1.1
45-IMMOB	1.5	1.4	1.1
46-LOY_IMP	1.5	1.4	1.1
47-SER_PROF	1.5	1.4	1.1
48-LOGICIELS	1.5	1.4	1.1
49-RetD	1.5	1.4	1.1
50-SER_ADMIN	1.5	1.4	1.1
51-SER_ENSEIG	1.5	1.4	1.1
52-SANTE_AS	1.5	1.4	1.1
53-ARTS	1.5	1.4	1.1
54-RESTO_HOTEL	1.5	1.4	1.1
55-AUT_SERV	1.5	1.4	1.1
56-VT_ISBL	1.5	1.4	1.1
57-VT_ADMPUB	1.5	1.4	1.1
58-SERV_ISBL	1.5	1.4	1.1
59-G_ENSEIGN	1.5	1.4	1.1
60-G_SANTE	1.5	1.4	1.1
61-G_AFED	1.5	1.4	1.1
62-G_APROV	1.5	1.4	1.1
63-G_AMUN	1.5	1.4	1.1
64-G_AAUT	1.5	1.4	1.1